Using Spatial Statistics and Geostatistical Analyst as Educational Tools



By Konrad Dramowicz Centre of Geographic Sciences Lawrencetown, Nova Scotia, Canada

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#### **Presentation Outline**

- Centre of Geographic Sciences
- Spatial Analysis and Geostatistics Course
- Learned skills
- Evolution of software tools used
- Student work samples
  - Prediction, error and probability surface maps



Point pattern analysis

- Spatial autocorrelation
- Hot spot analysis
- Measuring spatial distribution
- Geographically weighted regression

Located in Lawrencetown, Nova Scotia, Canada
Population: ~ 300

- Number of students: 170 180
- Number of COGS alumni currently working for ESRI: 50

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**Centre of Geographic Sciences** 

- Number of faculty: 18
- ESRI software user since 1983
  - GIS program since 1985

 New Advanced Diploma in Geographic Sciences with concentration in GIS, Remote Sensing or GIS for Business will be launched in 2010/2011

#### **Spatial Analysis and Geostatistics Course**

- The Spatial Analysis and Geostatistics course has been offered at COGS under different names since 1991
- The course has been delivered as an elective for students from GIS, GIS for Business, and Remote Sensing Advanced Diploma Programs
- The number of course participants has varied from 3
   to 12

#### **Spatial Data Analysis Module**

 Data Analysis module is mandatory for all students (30-45) taking the Advanced Diploma in Geographic Sciences program in their first term

 This module includes exploratory data analysis, geographically weighted regression, and some fundamental multivariate statistics in a spatial context



#### **Necessary Basic Statistical Skills**

- Using descriptive statistics
- Applying regression and curve fitting
- Understanding normal and t distributions
- Doing cluster analysis
- Comprehending statistical decision theory
  - Significance and confidence
    - Critical values
      - **Testing statistical hypotheses**

#### Using Statistical Packages with ArcGIS

- The following links were the most common for ESRI users who wanted to add more statistical functionality to GIS
  - SAS Bridge (ArcGIS)
  - S-Plus for ArcView (ver. 3.0 3.2)
- In this course, IBM PASW software package (formerly known as SPSS), was used and data were



passed to and from ArcGIS using an intermediate file format (\*.dbf)

Also, students learn how to perform statistical analysis using MS Excel (low cost solution)

#### **Course Learning Outcomes**

- The learners completing the Spatial Analysis and Geostatistics course demonstrate the ability to:
  - Select the most appropriate tool for solving a given problem
  - Map and interpret results
  - Think spatially while setting model parameters and interpreting results

Properly use fundamentals of statistics as a foundation for spatial analysis and geostatistics

#### **Course Textbooks**

- Mitchell Andy, 2005. The ESRI Guide to GIS Analysis; Volume 2: Spatial Measurements & Statistics, ESRI
- De Smith Michael J., Michael F. Goodchild, Paul A. Longley, 2009 (3<sup>rd</sup> edition). Geospatial Analysis: A Comprehensive Guide to Principles, Techniques and Software Tools, Matador



Johnston Kevin *et al*, 2001. Using ArcGIS Geostatistical Analyst, ESRI

### **Changes in Software Used**

- ArcInfo Workstation → ArcGIS
- GS+ → Geostatistical Analyst
- IDRISI → Spatial Statistics Toolset
- AML  $\rightarrow$  Python (with Tkinter)

Python

- SpaceStat → GeoDa / Spatial Statistics Toolset
- ArcView / Avenue (Wong & Lee scripts) → ArcGIS /



MapInfo / MapBasic  $\rightarrow$  ArcGIS / Python Hawth's Tools

Fragstats

#### Main ArcGIS Tools Used in the Course

- Geostatistical Analyst and tools
- Spatial Statistics Tools





#### **Recent Evolution of Geostatistical Tools**

- Geostatistical Analyst: ArcGIS 8.1, April 2001
- Geostatistical Wizard enhancements, nine new geostatistical tools: ArcGIS 9.2, September 2006
- Geostatistical Analyst supported by ArcGIS Server, animated tutorial: ArcGIS 9.3, June 2008
- Eleven new geoprocessing tools, improvements to the Geostatistical Analyst Wizard: ArcGIS 10.0, Fall 2010

## Recent Evolution of Spatial Statistics Tools

- ArcToolbox Window with eighteen Spatial Statistics tools (10.7% of all available tools: ArcGIS 9.0, May 2004
- Two more tools (including *Ripley's K Function*): ArcGIS 9.2, September 2006
- New Geographically Weighted Regression tool: ArcGIS 9.3, June 2007



New Ordinary Least Squares tool: ArcGIS 9.3.1, May 2009

One new and eight improved tools: ArcGIS 10.0, Fall 2010

#### Geostatistics: Basic Topics Taught

- Regionalized variable
- Deterministic vs. geostatistical interpolation techniques (algorithms, examples, advantages and disadvantages)
- Global and local trends, neighborhood search, anisotropy
- Kriging methods, semivariance models, cokriging
   Transformations, cross-validation, bivariate distribution, detrending, declustering
   Gaussian simulations
   Prediction, error, and probability maps

#### **Basic Properties of Regionalized Variable**

- Applied for continuous spatial distribution
- Its nature is mixed: somewhere between deterministic and stochastic
- Used for modeling very complex processes



#### Major Advantages of Using Kriging

- Exact interpolation
- Standard error maps come as a derivative
- Allows the user to control numerous parameters



## **Controlling Kriging Parameters**

- There are many kriging parameters to control and students are asked to control one parameter at time
- The optimal settings are those which minimize standard errors (root-mean-square, average standard error, or mean standardized)



#### Some Kriging Parameters to Control

- Kriging methods (7)
- Transformations (4)
- Semivariance models (7)
- Trend order (5)
- Anisotropy (individual values)
- Nugget (individual values)



Number of histogram lags and their size Neighborhood search (number of neighbors, minimal number, four search shapes)

#### Flow Chart of Order of Variables Considered



#### **Excel Tool for Teaching Kriging**

		Krigi	ng		
	Input	data			Dist
Wells	X	Y E	levation		3.35
1	3.00	4.00	120.00		2.88
2	5.30	3.40	103.00		3.35
3	2.00	1.30	142.00		4.79
ρ	3.00	3.00	2		2.88
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Wells	1	2	3	p	Kriging - dema
1	0.00	3.35	2.88	1.00	
2	3.35	0.00	4.79	3.32	Ealcolate distances
3	2.88	4.79	0.02	1.97	
					Calculate semivariances
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#### Critical Opinions About Geostatistics (1)

Phate by Nek Didleb

Jan W Merks

metrologist, author,

lecturer, consultant,

whistleblower, 'gadfly',

'pariah', 'iconoclast',

CIM Life Member



Hime

What's wrong with

Campling paradox

geostatistics?

Orged estimate

Minting variance

Iriaina Variance

**Geostatistical factoids** 

functional dependence

Spatial dependence

legrees of freedom

Interpolation or

Toletny Syndrome

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#### GEOSTATSCAM.com Benstatistics: from human orror to scientific froud

Or Fritz Agterberg, IAMG's Past President and NRCan's Emeritus Scientist, has not explained why he stripped the variance off his distance-weighted average point. orade in 1970 and in 1974.

Dr Roussos Dimitrakopoulos, McGill's miracle man and IMG's Editor-in-Chief, talked about stochastic mine planning optimization at APCOM 2009. He should apply it to Gemcom's gold resource for Busang and see what sort of mining plan his voodoo variances cook up.

What kept Bre-X humming along was to assume gold mineralization between saited boreholes, to knige and smooth a little, and to rig the rules of statistics a lot. The National Securities Regulator should rule that unbiased confidence limits for metal contents be reported.

Matheron, the self-made wigard of odd statistics, lost the variance of the lenghtweighted grade of core samples in 1954. and the variance of the length-weighted block grade in 19601

Or F.P. adjusting lost the variance of his distance-weighted average point. grade first in his 1970 paper and once more in his 1974 Geomathematics. He did refer to Fisher's F-test elsewhere in this textbook but didn't know how to test for spatial dependence between core samples in a borehole, or between boreholes on a line. He ought to delete Chapter 10 Stationary Random Variables and Kriging. How long does it take to right a wrong? Why does he assume that his distance-weighted average point grade needs no variance? It's high time to explain why?

In November 1989, we applied Fisher's F-test in Precision Estimates for Die mes to confirm spatial dependence between gold grades of ordered. rounds in a decline. Testing for spatial dependence troubled P CIM Bulletin's most dedicated enforcer of Matheronian geostatistics. Scores of similarly gifted geostatistocrats poetulated spatial dependence may be inferred unless proven otherwise. All sorts of degrees of freedom fighters were troubled when pevorg spatial dependence between gold grades of ordered rounds. Fisher's F-test. also proved the intrinsic variance of Bre-X's bogus gold in Busang's barren rock to be statistically identical to zero, as it ought to be. Sound statistics did so several months before Bre-X's boss salter vanished! Bre-X's original and duplicate bogus assays for a few early boreholes could have proved early on that a salting scan was in progress at the Busang project!

#### Geostatistics or voodoo science?

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#### by Jan Morks Special to The Northern Minor

The objective of mineral exploration is to find ore deposits, and to estimate grades and con-

tents in an unbi-Guest Column ased manner and with a reliable, realistic and affordable degree of preci-sion. The term "unbiased" implies that

the estimated grade and content of an ore deposit are statistically identical to its unknown true grade and content. A bias or systematic error in the measurement chain distorts the metal grade and content of an ore deposit.

Kriged variances are mathematically invalid and are useless for perphyry deposits, meaningless for stratabound deposits and even dangerous for gold deposits. Author M. David cautions that his

book, "Geostatistical Ore Reserve Es-timation," is not for professional statisticians and correctly predicts that they will find many unqualified statements

Kriging attempts to make a few drill holes go a long way. It creates ore grade values in domains where grades and variances cannot possibly be estimated with a realistic degree of confidence, and where discontinuities in mineralization are bound to occur. Thus kriging tends to inflate expecta-tions for the continuity of mineralization between measured data points

with often devastating consequences. Covariances and kriged variances dominate geostatistics. Yet dependencies and degrees of freedom, concepts even more fundamental than variances, seem irrelevant to the geostatistical theorist. Perhaps, not surprisingly, the geostatistical theorist likes to fit spherical, exponential or linear models. to the variance terms for ordered data sets. After all, continuous functions that obscure mundane matters such as dependencies and degrees of freedom most effectively, make a fitting choice!

Some geostatisticians seem cautiously concerned. For example, M. Armsteorg and N. Champigny, (CIM Bulletin, March 1989), display smooth plots of kriged variances planging to zero in chaotic domains. The authors caution in the abstract that "the oversmoothed estimates should not be used for calculating recoverable reserves" which is quite understandable.

Kriged variances converge to zero exponentially so that only a few fabricated data points are required to generate suspiciously low variances in small blocks. Zero vari-

ances, which permit perfectly precise even geostatisticians find hard to be-

In another paper, David, too, dis-plays his fondness for smoothing in "Grade control problem dilution and geostatistics: choosing the required quality and number of samples for rade control" he shows how to predict recoveries and grades by entering differences between estimated variances and kriged variances into smoothing relationships that some computer programs so conveniently provide.

The estimated and kriged variances turned out to be statistically identical which implies that their differences are random numbers. Applying mathe-matical analysis, or just a little smoothing, to that class of random numbers is an exercise in futility and a typical example of voodoo statistics of the worst kind. Nonetheless, recoveries are predicted with one decimal point and grades with two.

Kriged variances inflate expectations for the continuity of mineralization between measured data points and cannot possibly deliver the reli-able and realistic precision estimates that ore reserves demand. However, a technique that replaces a bulldozer's crooked blade with a surgeon's scalpel has already been developed and published

The "grade-squared partition technique" merges Matheronian basics with the conventional block design and proven statistical techniques that take full advantage of the additive property of variances for grades and contents. Not only does it provide reliable and realistic precision estimates but it is also an effective tool to optimize exploration programs. It would allow geologists to apply statistical tests and techniques in a spreadsheet environment on laptops and portables in the field, and combine geological in-terpretations and applied statistics most effectively.

Jan Merks is president of Matrix Consultants in Vancouver.

The Northern Miner April 20, 1992

Who's what and when? Statistical facts

Valiance of a set. Variance of a central **ishin** Degrass of freedom Counting degrees of free-dairs

Feat for spatial dependênciê

#### The Bre-X fraud

Satted boreholes Satted borsheles and kriged boreholes. intrinic varianca

#### Critical Opinions About Geostatistics (2)

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#### Geostatistics or voodoo science?

by Jan Morks Special to The Northern Miner

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## Student Work Samples: Prediction Surface Maps

- Driving time to Subways restaurants in Halifax <sup>1</sup>
- Driving time to Nova Scotia Breast Screening Program's clinics
- Prediction and standard error surfaces: number of houses sold per km sq in Halifax Regional Municipality in 2009
- Spending on unhealthy products

Number of recorded GPS locations of tourists in Kejimkujik National Park per one grid cell

<sup>1</sup> Only one item from the list of student work samples is provided with this document. However, all of them they will be demonstrated at a time of presentation

#### **Isochrones Mapping**



This assignment draws on Network Analyst, Geostatistical Analyst and geoprocessing using model builder to derive to distance models for Subway locations in Halifax. Through the Halifax street network, routes were calculated from every junction to the closest Subway location within the network. By calculating fields based on the solved network analyst layer, appropriate time and length values were derived. Ordinary kriging was then used to calculate the drive time and length

isochrone maps shown below.





## Student Work Samples: Probability Surface Maps

- Percentage of woman aged 40-60 receiving breast screening exceeds 20%
- Percentage of spending on unhealthy products exceeds 17%
- House prices exceeding \$150,000 in Halifax Regional Municipality 1982-1986
- House prices exceeding \$150,000 in Halifax Regional Municipality 1997-2001



## Spatial Analysis: Basic Topics (1)

- Point pattern analysis
  - Nearest neighbor analysis (Spatial Statistics tool)
  - Quadrat analysis (implemented by students using geoprocessing and scripting)
  - K and L functions (Spatial Statistics tool)
- Spatial autocorrelation
  - Moran (Spatial Statistics tool)



Geary (ArcInfo Grid)

Local Moran (Spatial Statistics tool)

## Spatial Analysis: Basic Topics (2)

- G statistic
  - General G statistic
  - Hot spot analysis
- Measuring geographic distributions
  - Mean center and central feature
  - Standard distance and standard deviational ellipse
  - Linear directional mean



#### Implementing Quadrat Analysis

- Use multiple point data sets
- Determine optimal quadrat size
- Calculate lambda (average number of points per quadrat), variance
- Use *t* statistic, various significance levels, determine point pattern
- Analyze relationships between quadrat size and point pattern

#### **Enhancing Nearest Neighbor Analysis**

- Use multiple point data sets
- Apply various neighborhood orders (1-6)
- Apply edge effect correction factor
- Analyze relationships between neighborhood order and z statistic
- Determine pattern for various significance levels



X	
Quadrat Method     Quadrat method represents the point pattern     analysis. This tool creates the fishnet composed of     quadrats (squares). Points are overlayed over     quadrats, the number of points per quadrat is     determined and the number of quadrats with a     particulat number of points is calculated. The     variance for random distribution is computed based     on the Poisson distribution. Then the test statistic t     is used for determining one of three patterns:     clustered, random (the benchmark) and regular. The     tool provides the output with all statistics, including     the confidence level.	
Tool Help	<u>_ 0 ×</u>
Point Data Set     Base Map     Neighborhood Order     I     Edge Effect	
	Quadrat Method         Quadrat method represents the point pattern analysis. This tool creates the fishnet composed of quadrats (squares). Points are overlayed over quadrats, the number of points per quadrat is determined and the number of quadrats with a particular number of points is calculated. The variance for random distribution is computed based on the Poisson distribution. Then the test statistic t is used for determining one of three patterns: clustered, random (the benchmark) and regular. The tool provides the output with all statistics, including the confidence level.         ▼       ▼         Tool Help       S         Neighborhood Order       1         I       ✓

## Student Work Samples: Point Pattern Analysis

- Spatial distribution of Nova Scotia Community
  - College students and campuses
- Spatial distribution of GPS locations representing location of tourists in Kejimkujik National Park



#### Distribution of Nova Scotia Community College Students

RESULTS OF OUADRAT ANALYSIS Point coverage: students Number of points: 7937 Applied quadrat size (m): 4829.43 Number of quadrats: 3201 Average number of points per quadrat: 2.48 Variance of the number of points per quadrat: 246.89 Confidence level: 0.95 Test t statistic: 9776.47 Critical t value: 1.96 Point pattern: CLUSTERED





#### **Spatial Autocorrelation**

 ArcInfo Workstation Grid commands Moran and Geary have two drawbacks:

- Work only for grid data
- Provide only values of coefficients which does not allow the user to make conclusion about the pattern (test statistics or significance levels are missing)
- ArcGIS does not have a tool for calculating Geary's coefficient of spatial autocorrelation c.

The tool for Moran coefficient / provides an appropriate output for making conclusion about pattern

#### ArcInfo Workstation Grid Commands Moran and Geary

Available GRIDs

GRID100 GRI D200 GRI D300 GRI D50 Grid: moran grid100 Running... 100% Moran's Coefficient (I): 0.8225915149909 Grid: geary grid100 Running... 100% Geary's Coefficient (c): 0.1323510913329 Grid: moran grid200 Running... 100% Moran's Coefficient (I): 0.6867757219644 Grid: geary grid200 Running... 100% Geary's Coefficient (c): 0.2319805102707 Grid: moran grid300 Running... 100% Moran's Coefficient (I): 0.5987939746177 Grid: geary grid300 Running... 100% Geary's Coefficient (c): 0.2877525626295 Grid: \_

GRI D400

### Spatial Autocorrelation (Global Moran's /)



# Student Work Samples: Spatial Autocorrelation

- Hurricane statistics (scale intensity, damage, injuries, fatalities, mean path width) for continental USA, central USA, and Kansas
- Patterns on spending on eleven food categories
   across Nova Scotia
- Patterns on fishing five species (cod, haddock, yellowtail, skate, lobster) in three fishing areas

(en are

(entire bank, western portions and selected areas of Georges Bank) near Nova Scotia

## Spatial Autocorrelation of Hurricanes for Three Study Areas and Five Variables

Study Area	Variable Name	Moran's I Statistic	Variable Variance	Test Statistic Z		Spatial Autocorrelation Pattern
Continental USA	Fujita Scale Intensity	0.348765	0.000304	20.019246		Positive
Continental USA	Damage in Dollars	0.390424	0.0003	22.544	983	Positive
Continental USA	Injuries	0.020646	0.000235	1.347	858	Positive
Continental USA	Fatalities	0.055808	0.000235	3.646	061	Positive
Continental USA	Mean path width of tornado, expressed in tenths of feet	0.135205	0.000455	6.342	105	Positive
Study Area	Variable Name	Moran's I Statistic	Variable Variance	Test Statistic Z	Spa Patt	tial Autocorrelation tern
Central USA	Fujita Scale Intensity	0.26338	0.000775	9.46343	Pos	itive
Central USA	Damage in Dollars	0.376513	0.000085	12.935778	Positive	
Central USA	Injuries	0.005448	0.00031	0.313422	No	Autocorrelation
Central USA	Fatalities	0.000032	0.000401	0.005183	No.	Autocorrelation
Central USA	Mean path width of tornado, expressed in tenths of feet	0.058775	0.001356	1.600326	Pos	itive

Study Area	Variable Name	Moran's I Statistic	Variable Variance	Test Statistic Z	Spatial Autocorrelation Pattern
Kansas	Fujita Scale Intensity	0.2324	0.0108	2.24	Positive
Kansas	Damage in Dollars	0.3721	0.0125	3.34	Positive
Kansas	Injuries	0.004566	0.003576	0.085184	No Autocorrelation
Kansas	Fatalities	0.001556	0.001571	0.052579	No Autocorrelation
Kansas	Mean path width of tornado, expressed in tenths of feet	0.081955	0.052537	0.363351	No Autocorrelation

# Student Work Samples: Mapping Spatial Autocorrelation Clusters

- Clusters of Nova Scotia Community College students by Dissemination Areas
- Clusters of GPS locations of tourists in Kejimkujik National Park





#### Hot Spot Analysis

- Hot spot analysis is one of the most informative and visually appealed tools
- It is available trough such Spatial Statistics tools as
  - Hot Spot Analysis (Getis-Ord Gi\*)
  - Cluster/Outlier Analysis with Rendering
  - Hot Spot Analysis with Rendering
  - Z score Rendering



Results can be visualized using thematic classified maps of: Input variable *Gi*\* values *Z* values

## Student Work Samples: Hot Spot Analysis

- Relationships between G statistic and distance threshold
- Clusters of tornado in Kansas
- Clusters of visitors in Kejimkujik National Park
- Concentration of locations where tourists stopped in Kejimkujik Seaside Adjunct National Park
- High and low participation rates for the Nova Scotia Breast Screening Program



High and low spending on healthy food categories in Nova Scotia

Clusters of high and low house prices in Halifax Regional Municipality in 2009



#### Measuring Geographic Distributions

- Set of tool relatively easy to use and providing visually appealed results
- Extremely useful for such operations as:
  - Determining study area
  - Comparing spatial distributions and analyzing changes
  - Identifying the most accessible feature or location
  - Analyzing orientation and direction of linear features



# Student Work Samples: Measuring Geographic Distributions

- Determining study area based on weighted deviational ellipse for Air Miles cardholders
- Directional distribution of GIS jobs in North America
- Mean centre and deviational ellipse: weekday vs. weekend tourists; morning, mid-day and evening tourists in Kejimkujik National Park
- Succession of mean center and deviational ellipses



for houses sold in Halifax Regional Municipality 1978-2006

Geographic distribution of students and campuses of the Nova Scotia Community College

#### Mean Centre of Activity and Standard Deviational Ellipses for Weekdays vs. Weekends using Jakes Landing Data



#### **Geographically Weighted Regression**

- Geographically Weighted Regression (GWR) developed in late 1970s, was added to ArcGIS as a new tool in 2007 and it is a great extension to the multiple linear regression added to ArcGIS two years later as the Ordinary Least Squares (OLS) tool
- OLS is a global-scale tool providing output for the entire study area, whereas GWR is a local-scale tool providing output for each GIS feature



GWR has many input parameters and offers a lot of opportunities for mapping numerous output data

# Comparing Results from ArcMap (OLS) and SPSS (Multiple Linear Regression)

Estimated	Residual	StdResid	PRE_1	RES_1	ZRE_1
62094.4000000000	9359.5700000000	0.46264800000	62094.42766	9359.57234	0.46265
62075.00000000000	-62075.0000000000	-3.0684000000	62075.04437	-62075.04437	-3.06840
69164.00000000000	4829.9700000000	0.23874800000	69164.02794	4829.97206	0.23875
63603.1000000000	7310.93000000000	0.36138300000	63603.07268	7310.92732	0.36138
54370.90000000000	12036.1000000000	0.59495100000	54370.88642	12036.11358	0.59495
102495.00000000000	125025.0000000000	6.1800300000	102495.44337	125024.55663	6.18003
77754.1000000000	11521.9000000000	0.56953300000	77754.10527	11521.89473	0.56953
56124.4000000000	20428.6000000000	1.00979000000	56124.44101	20428.55899	1.00979
45699.2000000000	5511.7800000000	0.2724500000	45699.22475	5511.77525	0.27245
52594.4000000000	27485.6000000000	1.3586300000	52594.36524	27485.63476	1.35863
66351.70000000000	1396.3300000000	0.06902140000	66351.66782	1396.33218	0.06902
51774.4000000000	6917.6400000000	0.34194200000	51774.36189	6917.63811	0.34194
82810.2000000000	14541.8000000000	0.71880800000	82810.20622	14541.79378	0.71881
57318.2000000000	-3399.1700000000	-0.16802200000	57318.16535	-3399.16535	-0.16802
73462.4000000000	21169.6000000000	1.0464300000	73462.36951	21169.63049	1.04643
83895.7000000000	31318.3000000000	1.5480800000	83895.67461	31318.32539	1.54808
82938.9000000000	518.1320000000	0.02561150000	82938.86760	518.13240	0.02561
82991.0000000000	21640.0000000000	1.06968000000	82990.98000	21640.02000	1.06968
57334.00000000000	-4383.0500000000	-0.21665600000	57334.04623	-4383.04623	-0.21666
71167.3000000000	2275.7400000000	0.11249100000	71167.25526	2275.74474	0.11249

SPSS

#### Possible Problems with Using GWR

- Relationships between predictors and target variable are not linear
- Correlations among predictors is detected (collinearity)
- Different patterns exist across study area (nonstationarity)
- Presence of data outliers can bias predicted values
- Model performance depends on the magnitude of the



target variable value (heteroskedasticity) Spatial clusters of over-estimated and underestimated predicted values exist (spatial autocorrelation of residuals)

Residuals are not normally distributed

#### **GWR Input Parameters to Control**

- Set of independent variables (predictors)
- Distance or number of neighbors
- Optionally, two kernel types
  - Fixed
  - Adaptive
- Optionally, two bandwidth methods

**Akaike Informative Criterion** 

**Cross Validation** 

**Optionally**, using other raster-related parameters

## Comparison of GWR Scenarios: An Example

Scenario	Kernel type	Bandwidth method	Distance	Number of neighbors
A	Fixed	Parameter	4000 meters	N/A
В	Fixed	Parameter	2000 meters	N/A
C	Adaptive	Parameter	N/A	30
D	Fixed	AIC	N/A	N/A
E	Fixed	CV	N/A	N/A
F (weighted by area	Adaptive	Parameter	N/A	30
G (weighted by population)	Adaptive	Parameter	N/A	30

# Preferred Values of GWR Input Parameters

Preferred values					
Low values	High values				
Residual squares	R2 (close to 1.0)				
Sigma	R2 adjusted (close to 1.0)				
AICc					



#### Finding The Best Model: An Example

Scenario	Residual squares	Effective number	Sigma	AICc	R2	R2 adjusted
А	44,594,620,522	30.95	15,120	5,012	0.760	0.723
В	8,519,128,670	49.60	11,779	2,468	0.933	0.880
С	8,519,128,670	49.60	11,779	2,468	0.933	0.880
D	100,432,304,465	16.53	17,814	7,464	0.640	0.623
E	100,432,304,465	16.53	17,814	7,464	0.640	0.623
F	814,654,793	34.21	4,128	1,649	0.949	0.898
G	3,926,076,424,856	42.81	269,158	2,763	0.903	0.827

The best model is scenario F (Adaptive kernel, 30 neighbors, weighted by area)

#### Possible Output Maps From GWR

- 1. Observed values (column Observed)
- 2. Predicted values (column Predicted)
- 3. Raw residuals (column Residual)
- 4. Standardized residuals (column StResid)
- 5. Coefficients of determination (column Local R2)
- 6. Standard error (column StError)

Collinearity (column Cond)

Weighted factor (column Weight)

#### **Final Thoughts**

- Learners through practicing spatial analysis and geostatistics with ArcGIS acquire new analytical and research skills which can be used for
  - Enhancing students assignments in other courses
  - Completing major research projects and capstone projects
  - Continuing their education toward the degree of Masters in Geomatics, joined program between the Nova Scotia

Community College and Acadia University (Wolfville, Nova Scotia)