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Databases and Algorithms to Determine the Boundary of Wyoming

Introduction

Wyoming was established from parts of the Dakota, Idaho and Utah Territories in 1868 (15Stat.L.178,Ch.235). It was a simple designation of latitude and longitude lines

> Commencing at the intersection of the twenty-seventh meridian of longitude west from Washington with the forty-fifth degree of north latitude, and running thence west to the thirty-fourth meridian of west longitude; thence south to the forty-first degree of north latitude; thence east to the twenty-seventh meridian of west longitude; thence north to the place of beginning

The boundary did not change in 1890 when Wyoming was admitted to the Union (26Stat.L.222,Ch.664). However, the legal boundary is the surveyed boundary (Brown, 1981), and the first surveys of the state boundary began in 1869 and continued through until 1882. Resurveys were completed in the early part of the 20th Century, but the basic survey points were established with the original surveys.

The size of Wyoming was first published by the Census Bureau in 1870 as 97,883 square miles (253,502 sq. km.). In the 1880 Census report the reported value was 97,575. It is not clear in either year whether these numbers refer to land area or total area (land and water), nor have we been able to discover the method of determination. However, in 1881 the Census Bureau adjusted the size of Wyoming to 97,890 square miles (253,520 sq. km.), using an equation due to Carpenter, presumably with the Bessel ellipsoid (Gannett, 1881). This value was used by the U.S.G.S. in their boundary reports and by the Census Bureau for each of the succeeding Censuses until 1904. Gannett (1904) adjusted the area to 97,914 sq. miles (253,582 sq. km.) by using the Carpenter equation with the Clarke 1866 ellipsoid instead of the Bessel. This value is the area that was reported by both the Census Bureau and the U.S.G.S. as the size of Wyoming until 1980. The Census Bureau in 1980 used a different method to delineate the boundary of Wyoming and produced an area of 97,809 sq. miles (253,310 sq. km.). The recalculation did not use the Carpenter equation with latitude and longitude lines, but GBF/DIME files derived from the surveyed boundary portrayed on 1:24,000 scale U.S.G.S. quadrangles. The 1990 Census calculation of the state boundary and size used the TIGER files based on digitized 1:100,000 topographic maps and produced a size of 97,819 square miles (253,336 sq. km.). The 2000 Census revised this value slightly to 97,813 square miles (253,321 sq. km.)

Problem

Given these changes in the size of Wyoming it is difficult to know which is the correct size. Brown (1981) states that the boundary should be based on the surveyed and monumented boundary. If that is the case, then none of the published areas for Wyoming are correct because they were based on different boundaries and paper map locations. Initially, the Census Bureau and U.S.G.S. used the basic latitude and longitude values as published in the laws that established the territory and eventually the state. The later versions of the Census Bureau boundary taken from the 1990 and 2000 TIGER files were extracted from either 1:100,000 or 1:24,000 topographic maps. These maps may represent the boundary but their accuracy is only as good as the mapping standards. Thus, any well defined point on the boundary at 1:100,000 could be displaced from its true position by as much as 83 feet, or on a 1:24,000 map by up to 20 feet. This could affect the area of the state by several square miles. So, what is the boundary of Wyoming and how will that translate into a more accurate measurement of its area? The typical GIS user does not question the source, accuracy, or limitations of the boundaries they use and this can have consequences in boundary determination, size, and fundamental administrative issues.

Methodology

To create Wyoming's digital boundary and generate a more accurate size for Wyoming a three stage process was followed. In phase one, a number of different databases had to be constructed and integrated. Further, a series of algorithms had to be employed or developed to adjust the databases to differences in datum. Using the framework from the 1983 Multipurpose Cadastral Framework report of the National Research Council, a basic assumption of this project is to use the monumented positions and the most accurate surveyed points as the bases for the boundary. This would entail collecting the surveyed points and adjusting them to current geodetic, ellipsoid, and datum standards. To accomplish this, several sources were used:

1. U.S. BLM Geographic Coordinate Data Base (GCDB) field survey data for Wyoming (validated)
2. U.S. BLM GCDB field survey data for Wyoming (non-validated)
3. U.S. BLM GCDB field survey data for neighboring states (validated)
4. U.S. BLM GCDB field survey data for neighboring states (non-validated)
5. U.S. Forest Service GPS points.
6. U.S.G.S. digital raster graphics.
7. N.G.S. HARN points

Phase two of the project was the calculation of the ellipsoid-based size of Wyoming. To accomplish this phase of the project three steps were completed. These steps are:

1. Calculate the size of the state from the digital boundary
2. Adjust the boundary for potential systematic location inaccuracy and calculate the size of the state
3. Adjust the boundary for random inaccuracy and calculate the size of the state

In the final phase of the project, the size of Wyoming was determined by incorporating the elevation of the state into the equation.

Boundary Construction
The original survey information came from the different surveys of the state (Van Zandt, 1976). The eastern boundary was done in 1869 by Oliver Chaffee, while the southern boundary was started in 1873 by Alonzo Richards and the western boundary in 1874 by Alonzo's brother, William. Rollin Reeves surveyed the northeastern boundary from northwestern Nebraska to southeastern Montana in 1877. Reeves started the northern boundary in 1878 and completed it in 1879; some corrections were made in 1882. Overall, it took more than 13 years to complete the survey boundary of Wyoming.

The original survey data followed the Manual of Boundary Survey (1855) established by the General Land Office (which eventually became part of the U.S. Bureau of Land Management). According to the Manual all latitude measurements were calculated from astronomical positions. Thus, they do not correspond to a ellipsoid or a datum.

The first step in creating a new boundary file for Wyoming was to use the U.S. BLM Geographic Coordinate Data Base (GCDB), constructed from the survey field notes in positioning township, range and section makers. This data base consists of Public Land Survey (PLSS) township, range, section and sub-section monument positions. The digital data files contain latitude/longitude positions of the monuments along with their UTM and State Plane coordinates and estimates of accuracy level. All positions are based on NAD1927 which were converted to NAD83. Along the boundary, the GCDB points are in two different files, one for each state, and usually the positions of corresponding points differ. Thus one could use the GCDB to construct two boundaries for Wyoming, one based on the Wyoming files, the other based on the files from neighboring states.

Some of the GCDB data files are "validated" by the BLM, while others are not (the validated files are more accurate than the others). In order to produce a single boundary file, we used validated Wyoming data whenever possible. If a neighboring state's file was validated while Wyoming's was not, we used the neighbor's data; if neither was validated, we used Wyoming's.

In some cases BLM GCDB data did not exist for either state, but data from the Forest Service was available, so we used it. In other cases, notably along the northwestern border within Yellowstone National Park, no numeric digital files exist: There we digitized the border from 1:24,000 U.S.G.S. digital raster graphics.

The GCDB data base contains information about the accuracy of each point. Table 1 details the errors in the different borders, ranging from a high of 627 ft. (191 m.) to a low of 0 ft. (0m.) along the southern boundary.

Table 1. Inaccuracy in the Calculated Wyoming Boundaries.

	Northern Boundary	Eastern Boundary	Southern Boundary	Western Boundary
Number of Points	1,785	2,447	2,512	1,525
Average Mean Square Error (ft.)	61	93	64	106
Maximum Error (ft.)	627	554	470	419
Minimum Error (ft.)	3	2	0	3
Stand Deviation (ft.)	87	109	50	91

The second step in the process was to incorporate geodetic control points from the National Geodetic Survey. A mixture of High Accuracy Reference Network points and geodetic control points from NGS were extracted from their control point data base on the web. A total of 9 HARN and control points were selected as boundary positions. They represent the four corners of the state, the intersections with surrounding states, and Wyoming milepost 123 on the southern border. All locations are B order control points.

By merging these data sets, the complete boundary for Wyoming was formed. Table 3. lists the type of data set, and the number and percentage of points.

Table 2. Coordinate Points for Wyoming Boundary.		
Source	No. of Points	Percent of Total
NGS HARN	9	0.1%
Validated GCDB	8,619	94.6%
Non-validated GCDB	390	4.3%
U.S. Forest Service	64	0.7%
Digitized DRG	23	0.3%
Other	5	0.1%
Total	9,110	100.0%

Area Calculations

Phase two of the project entailed calculating the area of Wyoming from the digital boundary. In step one of this phase the area was calculated using the 9,110 points in an Albers Equal-Area projection. This calculated area is 97,808 sq. miles (253,322 sq. km.) with NAD83 datum. However, this does not include any adjustment based on the inaccuracy of the digital boundary points. If the values for "average mean square error" in Table 1 are themselves averaged, the GCDB digital points have an inaccuracy of 79' (24m). Thus, in step two of this phase, a systematic adjust of increasing the boundary by 79' (24m) to all points either in latitude for north and south boundary points and longitude for east and west boundary points would increase the size of the state by 19 sq. miles (49 sq. km.), to total 97,827 sq. miles (253,371 sq. km.). Likewise, a systematic adjustment of decreasing the boundary by 79' (24m) to all points, would produce an area of 97,789 sq. miles (253,272 sq. km.). Incorporating the inaccuracy would produce an area of 97,808 sq. miles ±19 sq. miles (253,322 sq. km. ±49 sq. km.).

However, it is not believed that the error is systematic either as an increase or decrease, but that the error occurs randomly. In step three of this phase, a random scenario of adjustment was calculated for all boundary points and this simulation is calculated 1000 times and then an average adjustment was calculated. In this step of the phase, the area is 97,808 sq. miles ± .25 sq. miles (253,322 sq. km. ± .65 sq. km.).

By integrating and adjusting for positional error it has been determined that a systematic error can add or subtract 19 sq. miles (49 sq. km.) to the area of Wyoming. However, a random displacement of boundary points based on the positional error only produces a .25 sq. mile (.65 sq. km.) error.

Elevation Adjustment to Area Computation

The area of Wyoming given above is derived from an equation that places the state on an ellipsoid. However, the land mass of Wyoming is actually elevated above the ellipsoid an average of about a mile ranging from a low point of 3,125 ft. (953 m.) to the highest peak at 13,804 ft. (4,207 m.). To adjust for this height, 48 one-degree DEMs (3 arc-second resolution) of Wyoming or portions thereof were used.

The first step in this phase of the research required an algorithm to raise all of the 1,442,401 points to their assigned elevation. Each point was converted to an Earth-centered, Earth-fixed three dimensional Cartesian coordinate system, then they were grouped into a square "patch" of four points and the center location and elevation was calculated. The patch was divided into four triangles and the area of each triangle calculated – it is easier to calculate the area of a 3D triangle than that of a 3D quadrilateral.

The second step was to project the (usually) tilted surface of each triangle onto a level plane at the average elevation of the triangle and to calculate its area. This gives the planimetric area of the triangle. This was repeated for the four triangles in each patch and for all1,440,000 patches for each DEM. After all of the patch areas were summed, the overall area was calculated to be 97,869 sq. miles (253,465 sq. km.). This is an increase of 60 sq. miles over the ellipsoid area. It should be emphasized that the area so found is **not** a slope area, but a **planimetric** area adjusted for elevation. In fact if Wyoming were perfectly flat, but elevated to its average height, it would still be 60 square miles larger than calculated by its ellipsoid area.

Conclusions

There are three conclusions that can be made from this research. First, the boundary of a state may be constructed by a multitude of sources; however, the basic assumption of the compilation should be to use the most accurate boundary position coordinates, not just mapped coordinates. Second, the area measurement should be identified as the ellipsoid planar area, which does not represent the true area of the state. And third, the elevated area is the most compatible to the surveyed and monumented border locations. This is critical because it is the surveyed areas that are used by local government for tax and property purposes.

The differences can manifest themselves in boundary disputes between states and counties, and complications for tax assessments and revenues based on errors in area determinations. For example, the 60 square miles added to the area of Wyoming is an adjustment of over $300 million in assessed value, representing about $22 million in tax revenues.

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