



# Curve Fit:

a pixel level raster regression tool

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# Curve Fit: a pixel level raster regression tool



- Working with multiple raster datasets that share a common 'theme' is becoming more common in landscape ecology and geospatial science.
  - Multiple images taken over time
  - Multiple images that represent spatial patterns measured at different scales
  - Use of multiple maps in spatial-temporal simulation models.
- We developed an ArcMap tool that allows users to model and visualize the changes that take place across multiple images over time and/or space.
  - Replaces multiple images with a few images of regression parameter estimates that characterize change across the full-range of images.

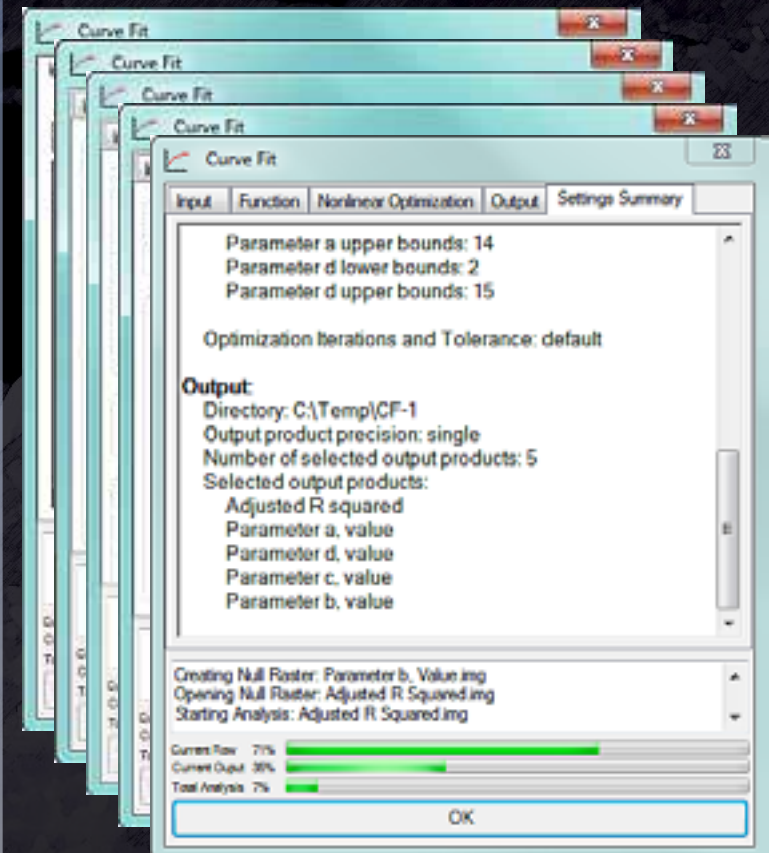


# Curve Fit: a pixel level raster regression tool



- Performs linear and non-linear regression analyses on raster datasets
- Outputs raster surfaces representing
  - Model solution
  - Model fit
  - Error estimates
  - Test statistics
  - Multi model inference criterion
- De Jager, N.R. and T. J. Fox. 2013. Curve Fit: a pixel-level raster regression tool for mapping spatial patterns. *Methods in Ecology and Evolution*, 4: 789–792.

Curve Fit Interface

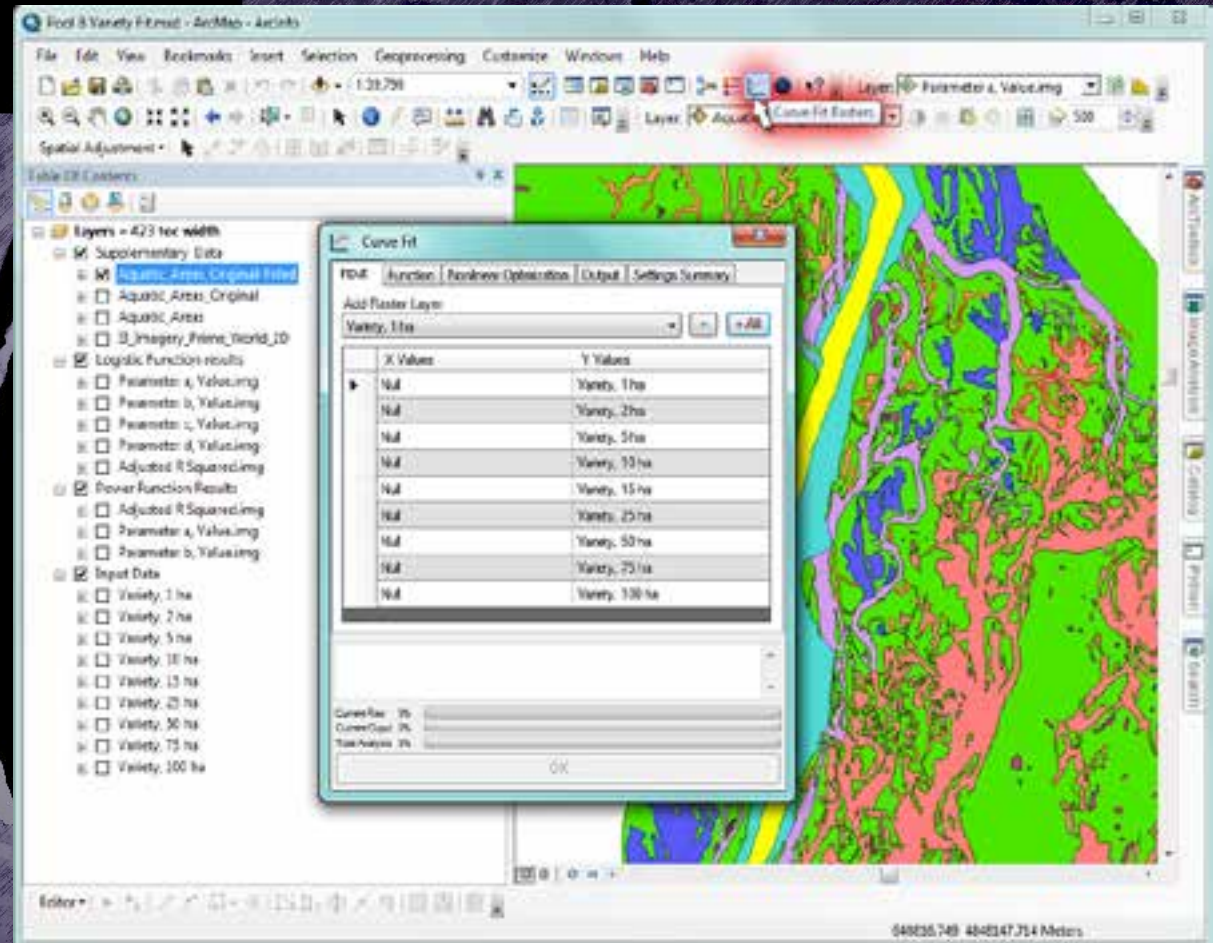




# Curve Fit: a pixel level raster regression tool



- Installs as an ArcMap 10.2 command
- Requires Spatial Analyst
- Utilizes the Extreme Math Optimization libraries





# Curve Fit dialog settings: inputs



- $X$  values
  - User enters an array of numeric  $X$  values
- $Y$  values
  - Raster layers
  - One raster for each  $X$  element
  - Missing value = No Data
- Example input datasets
  - Habitat density  $fn$  scale
  - Water depth  $fn$  dam discharge

Input data

	X Values	Y Values
▶	Null	Variety, 1 ha
	Null	Variety, 2 ha
	Null	Variety, 5 ha
	Null	Variety, 10 ha
	Null	Variety, 15 ha
	Null	Variety, 25 ha
	Null	Variety, 50 ha
	Null	Variety, 75 ha
	Null	Variety, 100 ha

Current Row: 0%  
Current Output: 0%  
Total Analysis: 0%

OK

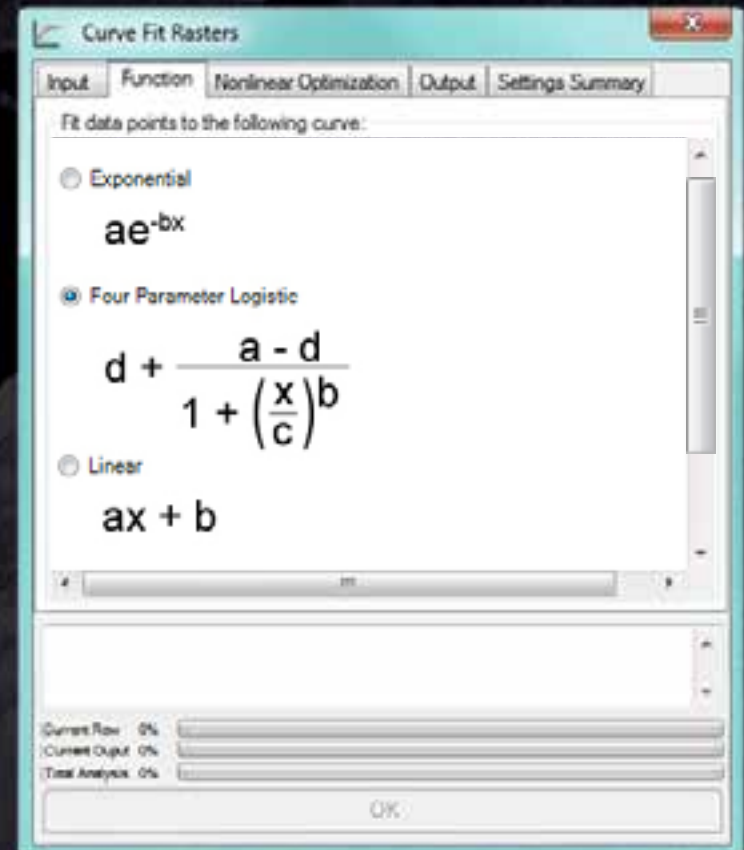


# Curve Fit dialog settings: functions



- Linear regressions
  - Linear
  - Specify degree polynomial
- Nonlinear regressions
  - Exponential
  - Four parameter logistic (4PL)
  - Power
- Functions can be amended
  - Write a custom class
  - Using inheritable base classes from the Extreme Math Optimization library
  - Plans to add an asymmetric sigmoidal curve

## Functions





# Curve Fit dialog settings: nonlinear optimizations



- Parameters
  - Constraints
  - Initial values
- Iterations
  - Minimum
  - Maximum
- Tolerances
  - Absolute
  - Relative
  - Require convergence
  - Convergence criteria
    - Within absolute tolerance
    - Within relative tolerance
    - Within either

## Nonlinear Optimization

The screenshot shows the 'Curve Fit' dialog box with the 'Nonlinear Optimization' tab selected. The 'Parameter Constraints and Initial Values' table lists parameters a, b, c, and d with their respective bounds and initial values. The 'Optimization Iterations' section shows 'Min' and 'Max' set to 'Default'. The 'Optimization Tolerance' section shows 'Absolute Tolerance' and 'Relative Tolerance' set to 'Default', and 'Convergence Criteria' set to 'Default'. The 'Current Rate', 'Current Output', and 'Total Analysis' progress bars are all at 0%.

Name	Lower Bound	Upper Bound	Initial Value
a	1	Default	Default
b	Default	Default	Default
c	Default	Default	Default
d	Default	14	Default

Optimization Iterations: Min: Default, Max: Default

Optimization Tolerance: Absolute Tolerance: Default, Relative Tolerance: Default, Convergence Criteria: Default

Current Rate: 0%, Current Output: 0%, Total Analysis: 0%

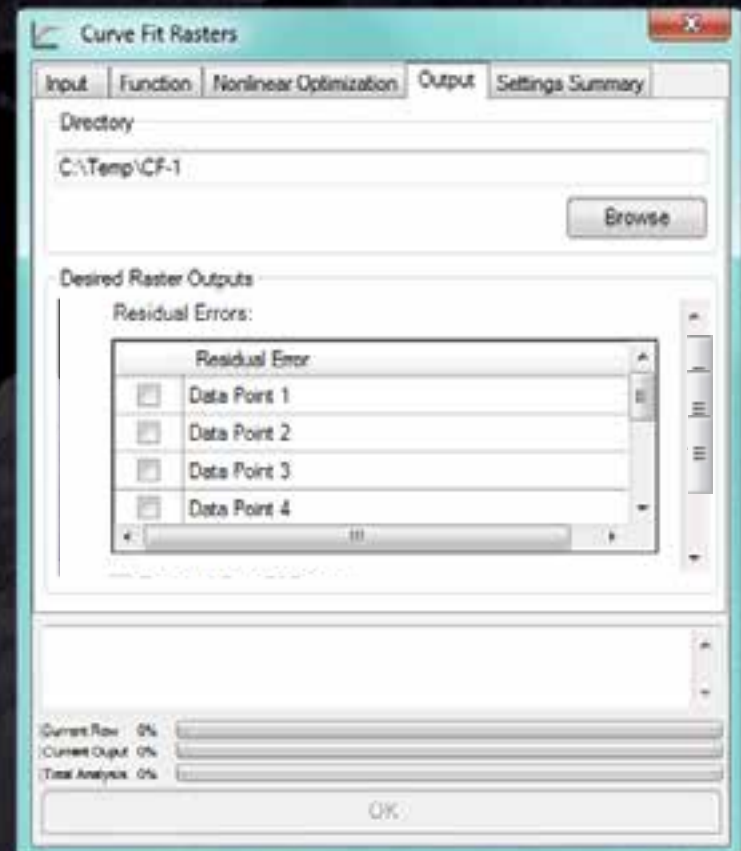


# Curve Fit dialog settings: outputs



- Output location
- No data value
- Outputs are stored as:
  - Single or double precision floating point IMG files
  - Double precision creates very large output files
- Frugal selecting outputs

## Output Products



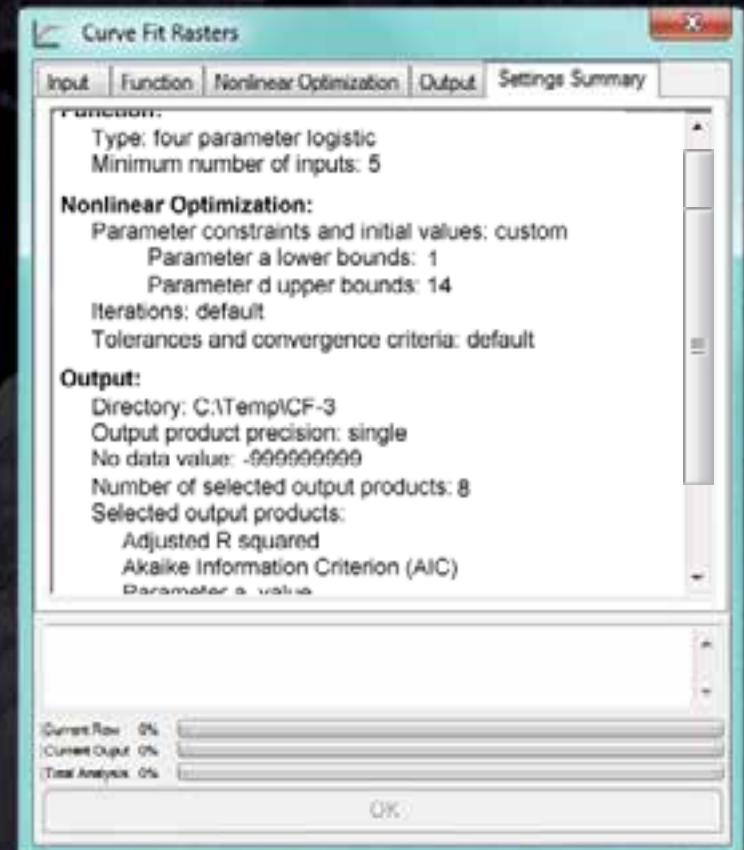


# Curve Fit dialog settings: summarization



- Settings checking
  - Minimum number of raster inputs determined by degrees of freedom
  - No missing  $X$  values
  - Output location
- Setting summaries, red means something is missing
- When all requirements are met the OK button is enabled

## Summary





# Curve Fit: model encapsulation



## Giving Diverse Models a Common Interface

- Physical process
- Authors
- Platforms
- Diverse proprietary software packages
- As long as there is a XY relationship Curve Fit provides a common solution





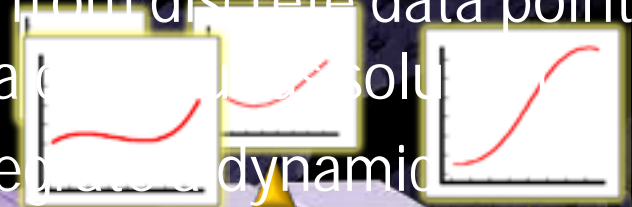
# Curve Fit: applicability



Curve Fit Derived Model



- Analytical applications (e.g. habitat variety *fn* scale)
- Encapsulate a bivariate process
- Go from discrete data points to a continuous solution
- Integrate a dynamic representation of the data series



Data Series

e.g. bathymetry,  
current velocity,  
shear stress

Parameter Estimates  
(coefficients)

— user queries a set of conditions  
— Curve Fit solution generates the response surface

**3<sup>rd</sup> Degree Polynomial**

$$a_3X^3 + a_2X^2 + a_1X + a_0$$



# Curve Fit: applications



- Characterizing spatial/temporal change:
  - Quantifying and mapping spatial patterns in habitat diversity across multiple scales (De Jager and Fox 2013).
  - Examining temporal changes in snow cover with respect to climate change (Lindsay et al. In prep).
- Use in spatial simulation models
  - Examining spatial patterns of river-floodplain connectivity (De Jager, Fox and Rohweder In prep).

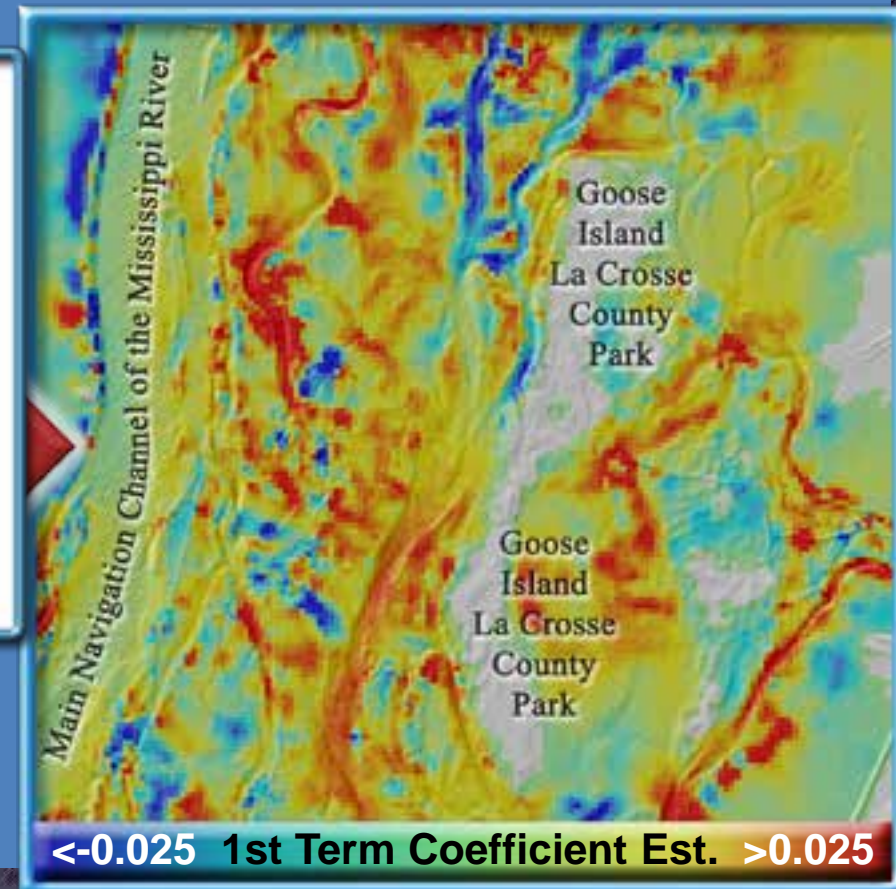
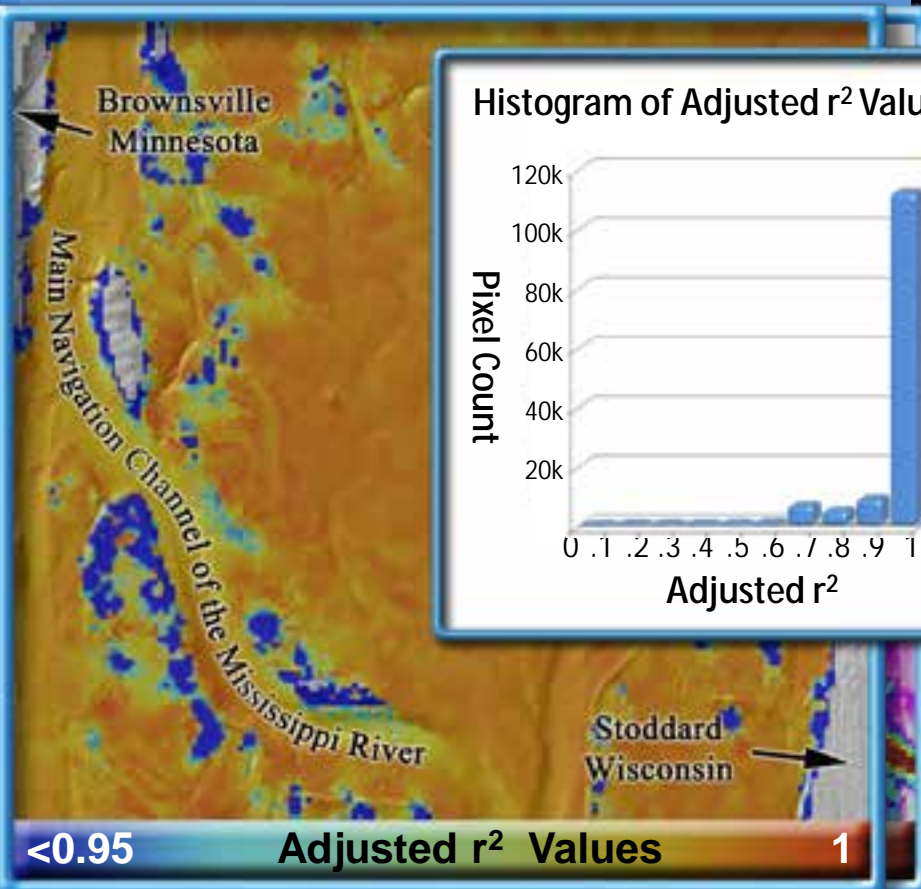


# Curve Fit: example output



Adjusted  $r^2$  at 10,000 Degree Intervals  
(Polynomial Models)

1<sup>st</sup> Term Coefficient Estimate, 3<sup>rd</sup>  
Degree Polynomial (1 of 4 coefficients)



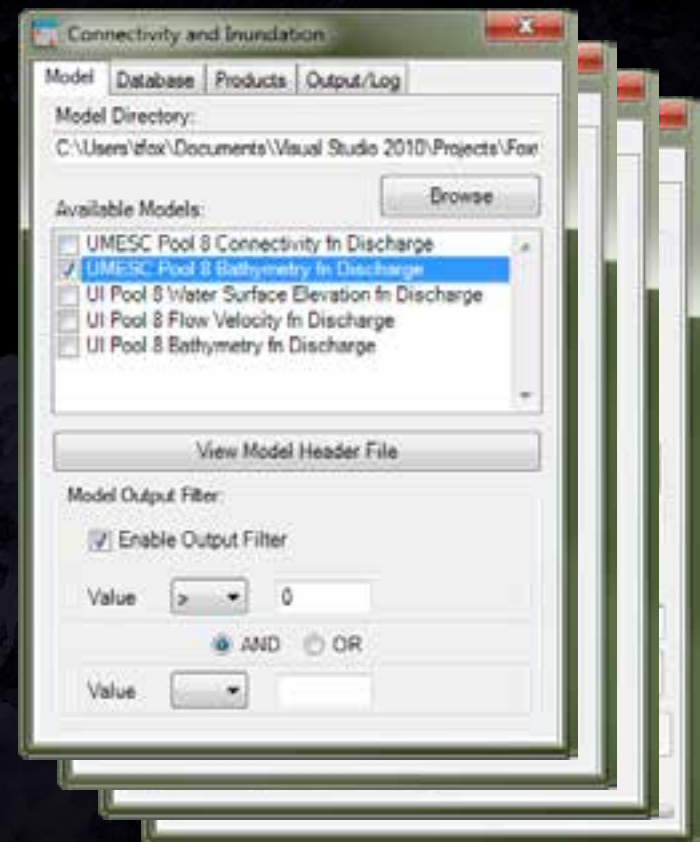


# Curve Fit: Connectivity and Inundation Tool



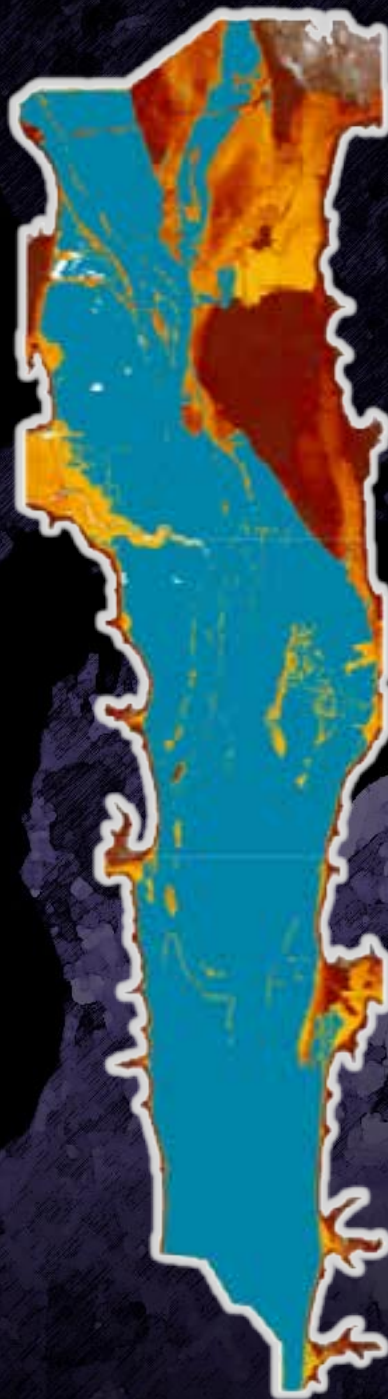
- Provides an expandable framework through the use of Curve Fit regression models
  - Curve Fit parameter estimates
  - XML header file
- Preloaded Curve Fit regression models
  - Bathymetry
  - Main channel connectivity
  - Flow velocity
  - Shear stress

## Connectivity & Inundation Interface





Moving from  
static maps  
to  
dynamic  
simulations





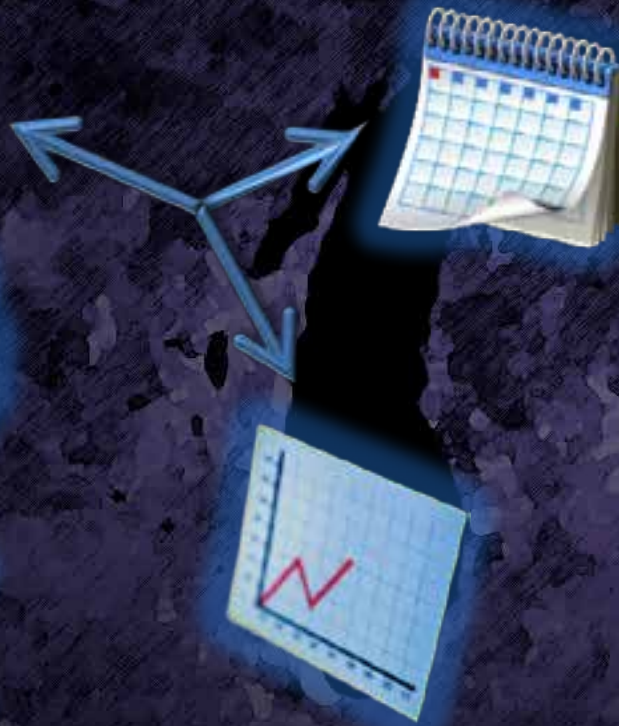
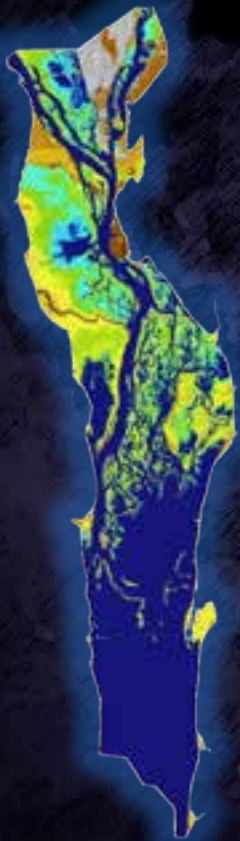
# Curve Fit: Connectivity and Inundation Tool



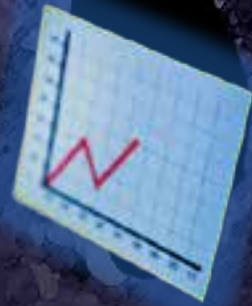
Curve Fit hydro model

DB of historic  
discharge rates

- Connectivity and inundation Tool
  - Database of discharge rates
  - User defined query
    - date range
    - model value range
- Applications
  - Vegetative succession
  - Fish distribution
  - Nutrient cycling and export



theoretical  
hydrograph



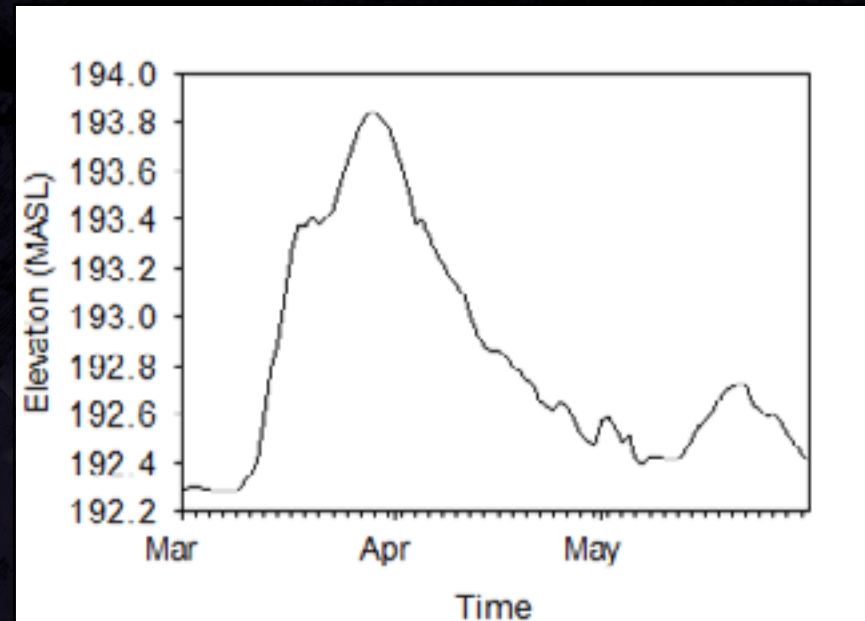


# Curve Fit: Connectivity and Inundation Tool



- Example output:
  - Curve Fit model:  
water depth *fn* dam  
discharge
  - Spring 2010 (March  
1<sup>st</sup> – May 31<sup>st</sup>)
  - Duration days where  
water depth > 0
- Strong flood that Spring
- Levee effects

Hydrograph, Spring of 2010



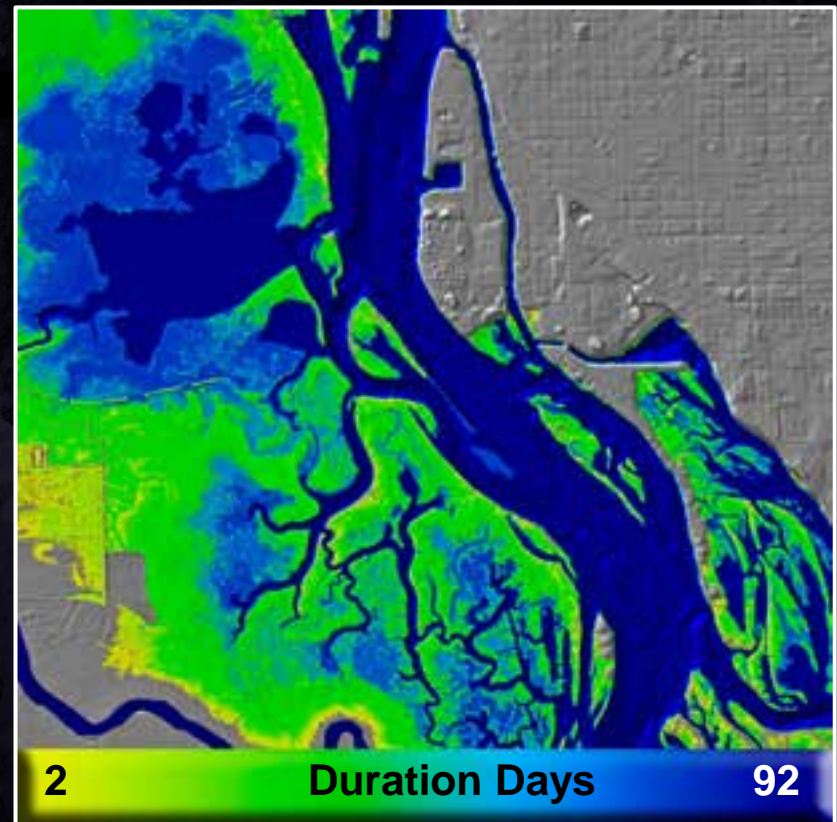


# Curve Fit: Connectivity and Inundation Tool



- Example output:
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  - Duration days where  
water depth  $> 0$
- Strong flood that Spring
- Levee effects

Main Channel Connectivity, Spring of 2010



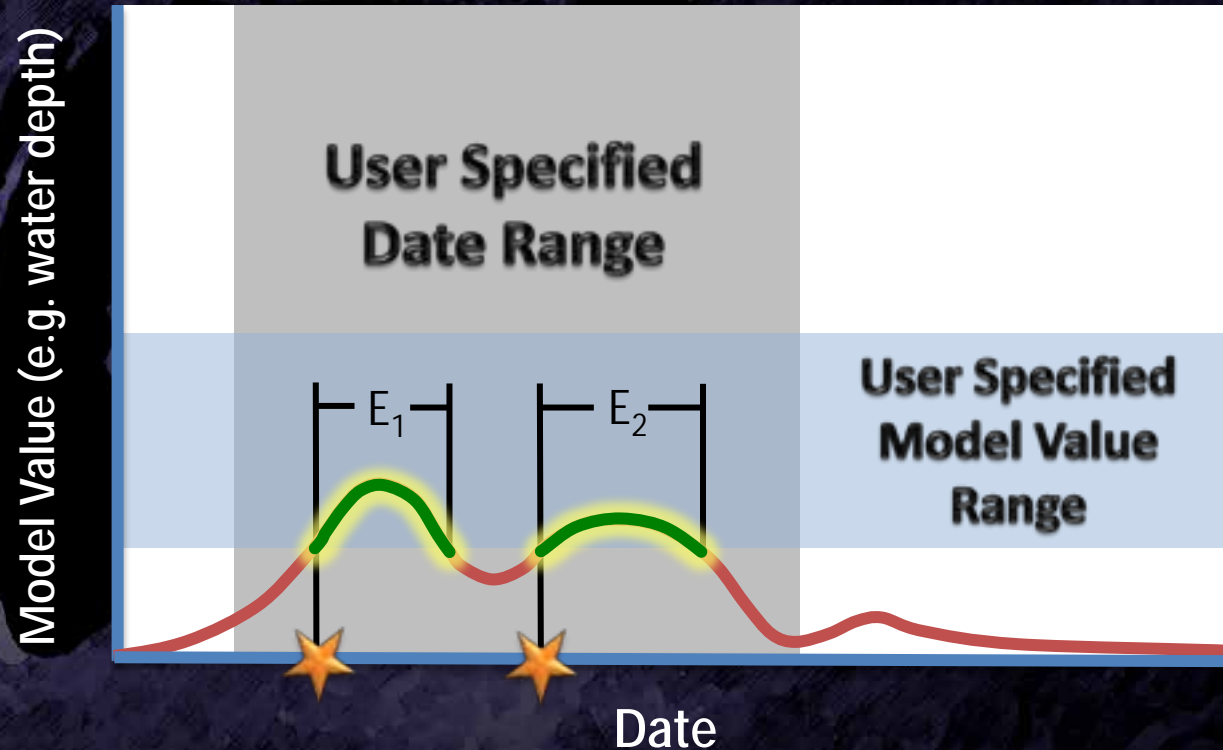


# Curve Fit: Connectivity and Inundation Tool



- User specified:
  - Model
  - Date range
  - Model value range
- Output products:
  - Event count
  - Duration days
  - Event start date
  - Summary statistics examples
    - Max flow velocity
    - Mean water depth

Model value over time with user specified ranges and resulting events

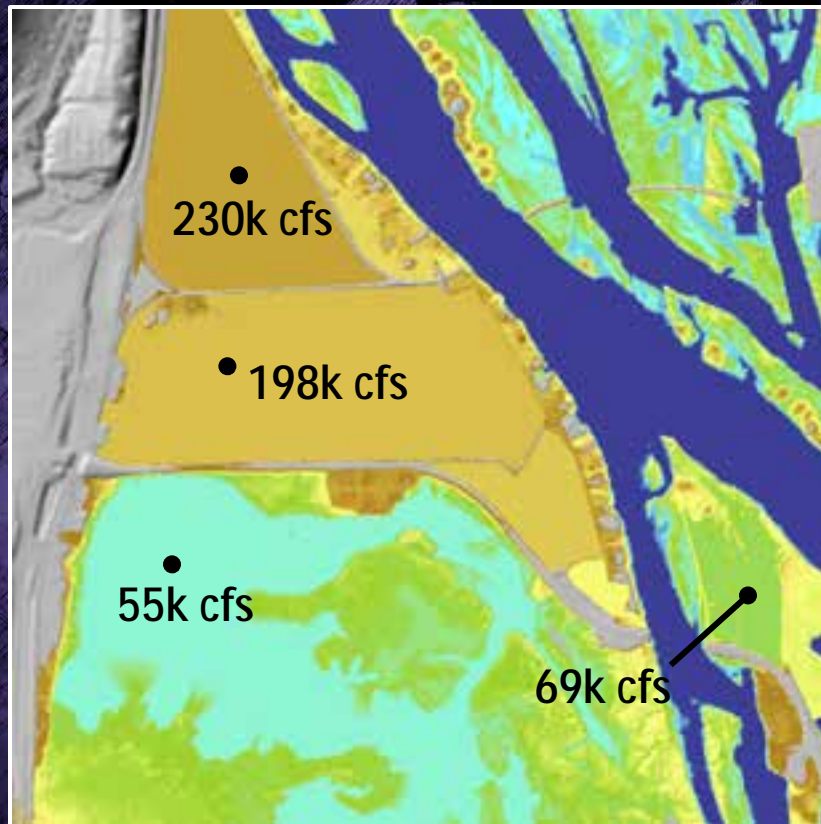




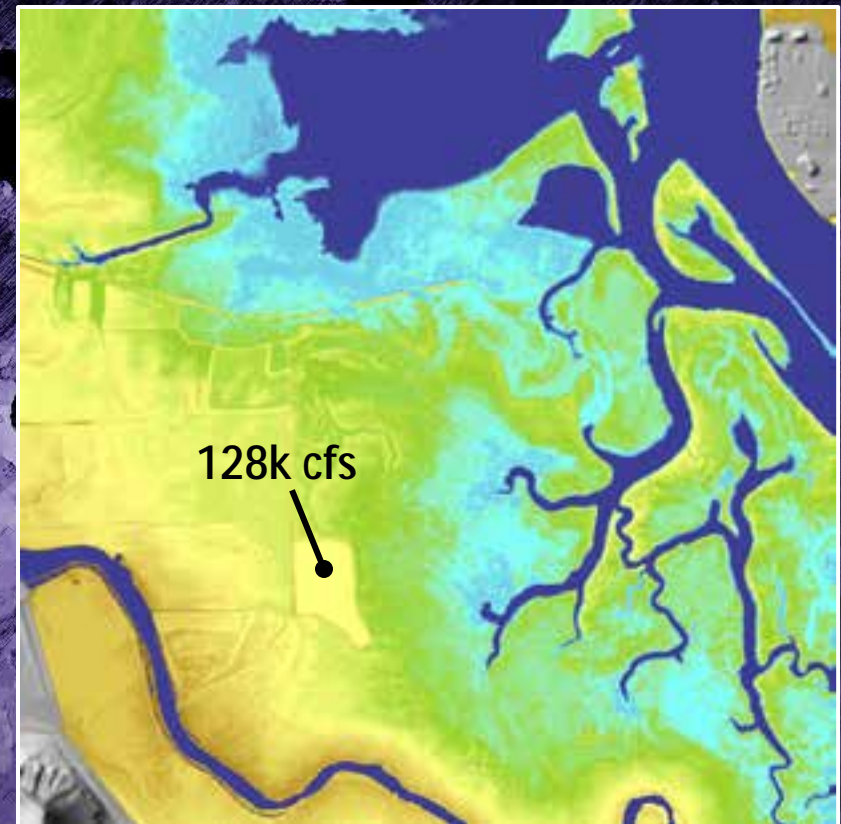
# Curve Fit: Connectivity and Inundation Tool



Blue Lake & Areas North



Target Lake & Areas South





# Curve Fit: Summary



- Robust analytical tool
- Performs both linear and nonlinear regression analyses on bivariate data
- Convenient method of capturing a continuous mathematical model from a discrete series of data
- Curve Fit polynomial regression models are used at UMESC to create flexible DSS tools

