Understanding Errors
In Measurements

James A. Coan, Sr., P.L.S.
A Friendly Pre-Test

1) Precision and Accuracy are the same thing  T  F
2) All errors can be avoided  T  F
3) All errors can be eliminated  T  F
4) How many significant figures will an answer have if a measured distance is multiplied by Pi?  
5) How many significant figures is 0.032?
6) Individual random errors are added together to find the total random error of a measurement  T  F
7) If a manufacturer states that your EDM can measure to ± (3mm+2ppm) what confidence level are they referring to?
8) An instrument centering error refers to how well a surveyor can set up over a point  T  F
9) In Surveying, all angle measurements are about the same precision  T  F
10) Weighting measurements allows the surveyor to put the error where it belongs  T  F
Understanding Errors In Measurements
(WHAT WE WILL COVER)

Errors and Mistakes
Precision and Accuracy
Sources of Errors in Surveying
Significant Figures
Random Error Propagation
Statistics for Land Surveyors
Random Errors in Angles and Distance
Practical Weights of Observations
Errors And Mistakes
Errors And Mistakes

Errors

The difference between the true and measured value of a measurement

The error equals the measured distance minus the true distance

E = M - T

They are unavoidable
Errors And Mistakes

Mistakes (Blunder)

A mistake is a blunder caused by carelessness

This type of problem has nothing to do with errors
Systematic And Random Errors
Systematic And Random Errors

Systematic Errors

Their magnitude and direction can be determined

They are predictable

Systematic errors can be eliminated

This separates them from random errors
Systematic And Random Errors

Random Errors
Are unavoidable
Can be minimized but never eliminated
Have a tendency to cancel but never completely do so.

Can be dealt with by the science of propagation
Precision And Accuracy
Precision And Accuracy

Precision

The agreement of readings of the same quantity

The better the precision the smaller the random error

Good precision exposes random error

Crude precision hides random errors
Accuracy

The agreement of readings with the true value

Accuracy is to systematic errors as precision is to random errors
Precision And Accuracy

- Precision
- Accuracy
Sources Of Errors In Surveying
Sources Of Errors In Surveying

Natural Errors

Caused by nature, wind, temperature, earth curvature, etc.

If the error is subject to known physical laws, it is systematic.

If the error does not follow known physical laws it is probably random.
Sources Of Errors In Surveying

Instrumental Errors

Caused by manufacture, wear and tear, or maladjustment of instruments.

This type of error can also be thought of as a blunder.

Most instrumental errors are random in nature.
Sources Of Errors In Surveying

Personal Errors

*Caused by the inability of a person perceive anything exactly*

*Can be controlled by good training, good motivation, and good technique*

*Personal errors are random and each person has their own*
Significant Figures
Significant figures is a part of mathematics that deals with the relevancy of digits in a number, and are mainly used in the surveying, scientific, and engineering community.
Significant figures can be separated into two areas:

1) Significant figures in measurements

2) Significant figures in computations
Significant Figures

Measurements

*Relates to the way measurements are made and recorded*

*No measurement is exact*

*When measuring, record only the digits that have meaning*
Significant Figures

Computations

Concerns itself with round-off error

The surveyor must use established rules of significant figures
Significant Figures

Rules

The following rules apply to both recording data and interpreting recorded data.
Significant Figures

Zeroes used merely to indicate the position of a decimal point are not significant

Example:

0.056 has two significant figures
Significant Figures

Rules

Zeroes recorded at the end of measurement are significant

Example:

1.30 has three significant figures
Significant Figures

Example:

Zeroes between non zero digits are significant

Example:

1.04 has three significant figures
Significant Figures

Rules

Numbers ending with one or more zeroes to the left of the decimal should have a special indication.
Significant Figures

Example:

175,000 has three significant figures

375,000 can have six significant figures
Significant Figures

Rules

When adding or subtracting measured distances, the number with the fewest decimal places will dictate the number of significant figures.
Significant Figures

Rules

Example: Adding or Subtracting

15.495

10.21

12.2 \text{ control, fewest number of decimals}

37.905

The proper answer is 37.9, having three significant figures
Significant Figures

Rules: Multiplication or Division

The product or quotient is determined by the fewest number of significant figures in the values used, if both are measured values.
Significant Figures

Example:

5.29 \times 0.052 = 0.28 \text{ two significant figures}
Significant Figures

Rules
Conversion factors do not determine significant figures

Example:
1534.5 in / 12 in per ft = 127.88 ft.
Significant Figures

Rules

With large or infinite number conversion factors; use one extra digit

Example:

174.35 x 3.14 = 547.46 ← incorrect
174.35 x 3.14159 = 547.74 ← correct
Significant Figures

Rules

With intermediate calculations use one extra digit and round off your answer

Example:

\[
\frac{43.56 \times 23.43}{21.02} = \frac{1020.6}{21.02} = 48.55
\]
Random Error

Propagation
Random Error Propagation

All measurements have random errors

Random errors have a tendency to cancel
but never completely do so
How random errors accumulate, cancel, decrease or behave through the process of computing the final value is termed “Propagation of Random Errors”
Random Error Propagation

Errors in a sum

Used when all errors are different

It is the square root of the sum of the squares of the errors

\[ E = \pm \sqrt{e_1^2 + e_2^2 + e_3^2 + \ldots + e_n^2} \]
Random Error Propagation

Errors in a Series

Used when the same error happens several times

Derived from the formula of errors in a sum

\[ E = \pm e \sqrt{n} \]
Random Error Propagation

Errors in a Product

Used in determining the random error of area calculations

Is also the square root of the sum of the squares of the errors

\[ E = \pm \sqrt{(Le_w)^2 + (We_L)^2} \]
Statistics For Surveyors
Statistics For Surveyors

Definitions

Direct Measurements

A measurement made directly between two or more points
Definitions

Indirect Measurements

A computed measurement between points
Definitions

Sample Size

The number of observations or measurements in a sample

\[ n = \text{Sample Size} \]
Mean

The sum of observations of a sample divided by the sample size

\[ \bar{X} = \frac{\sum X_i}{n} \]

\( \bar{X} \) = Mean  \( X_i \) = The value of the sample
Sample Size = 25

\[ \Sigma n = 779.4 \]

Mean = \[ \frac{779.4}{25} = 31.2 \]
Statistics For Surveyors

Definitions

Median

The middle value of the sample when the data is arranged in ascending or descending order.
Statistics For Surveyors

Definitions

Mode

The value which occurs most frequently in a sample
Theodolite Micrometer Readings
WILD T-2

<table>
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<th>Observations</th>
<th>Readings, Seconds</th>
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<td>24</td>
<td>32.3</td>
</tr>
<tr>
<td>25</td>
<td>32.7</td>
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</tbody>
</table>

Sum = 779.4
Mean = 31.2
Median = 31.1
Mode = 31.0 and 31.4
Range = 2.5
n = 25
Statistics For Surveyors

Definitions

Residual

The difference between an individual value in a sample and the mean of the sample

\[ V_i = \bar{X} - X \]

\[ V_i = \text{Residual} \]
Definitions

Standard Deviation

68% probability of an occurrence

\[
\sigma = \pm \sqrt{\frac{\sum v_i^2}{n-1}}
\]

\(\sigma = \text{Standard Deviation}\)
### Calculation of Standard Deviation

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<td>31.7</td>
<td>31.2</td>
<td>0.5</td>
<td>0.25</td>
</tr>
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</table>

\[ \sigma = \pm \sqrt{\frac{\sum V^2}{n-1}} = \pm \sqrt{\frac{10.04}{24}} = \pm \sqrt{0.418} = \pm 0.65 \]
# Statistics For Surveyors

## Levels Of Certainty

<table>
<thead>
<tr>
<th>NAME OF ERROR</th>
<th>SYMBOL</th>
<th>VALUE</th>
<th>% CERTAINTY</th>
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<tbody>
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<td>PROBABLE</td>
<td>$E_{50}$</td>
<td>0.6745$\sigma$</td>
<td>50</td>
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<tr>
<td>STANDARD DEVIATION</td>
<td>$\sigma$</td>
<td>1$\sigma$</td>
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<td>90% ERROR</td>
<td>$E_{90}$</td>
<td>1.6447$\sigma$</td>
<td>90</td>
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<tr>
<td>TWO SIGMA</td>
<td>$E_{95}$</td>
<td>2$\sigma$</td>
<td>95</td>
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<tr>
<td>99% ERROR</td>
<td>$E_{99}$</td>
<td>2.5$\sigma$</td>
<td>99</td>
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<tr>
<td>THREE SIGMA</td>
<td>$E_{99.7}$</td>
<td>3$\sigma$</td>
<td>99.7</td>
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</table>
Bell Curve

- 0.1% at 55
- 2% at 70
- 14% at 85
- 34% at 100 (mean)
- 34% at 115
- 14% at 130
- 2% at 145
- 0.1% at 145

68% between 85 and 115
95% between 70 and 130
Definitions

Standard Error of the Mean

The interval of uncertainty around the true value.

\[ \Sigma_x = \pm \frac{\sigma}{\sqrt{n}} \]
Definition: Standard Error of the Mean

\[
\sigma_{\bar{x}} = \pm \frac{\sigma}{\sqrt{n}} = \pm \frac{0.65}{\sqrt{25}} = \pm 0.13
\]
Pre-analysis

Formulas
Pre-analysis Formulas

**Reading Error**

- $\sigma_{\alpha_r} = \pm \frac{\sigma_r \sqrt{2}}{\sqrt{n}}$

- $\sigma_{\alpha_r} = \text{The total reading error}$
- $\sigma_r = \text{The individual reading error}$
- $n = \text{The number of angles turned}$
Pre-analysis Formulas

Reading Error

\[ \sigma_{\alpha_r} = \pm \frac{\sigma_r \sqrt{2}}{n} \]

- \( \sigma_{\alpha_r} \) = The total reading error
- \( \sigma_r \) = The individual reading error
- \( n \) = The number of angles turned

Repetition Theodolite

\[ \sigma_{\alpha} = \pm \frac{\sigma_r \sqrt{2}}{n} \]
Reading Error Example

Directional Theodolite

\[ \sigma_r = \text{individual reading error (0.65)} \]
\[ n = \text{number of angles turned (4)} \]

\[ \sigma_{\alpha_r} = \pm \frac{\sigma_r \sqrt{2}}{\sqrt{n}} \]

\[ \sigma_{\alpha_r} = \pm \frac{0.65 \sqrt{2}}{\sqrt{4}} = 0.46 \]
Reading Error Example

Repetition Theodolite

\[ \sigma_r = \text{individual reading error} \ (0.65) \]

\[ n = \text{number of angles turned} \ (4) \]

\[ \sigma_{\alpha_r} = \pm \frac{\sigma_r \sqrt{2}}{n} \]

\[ \sigma_{\alpha_r} = \pm \frac{0.65 \sqrt{2}}{4} = 0.23 \]
Reading Error Example

Topcon IS

18 SPECIFICATIONS

Laser class for distance measurement:
- Class 1 (IEC Publication 825)
- Class I (FDA/BHR 21 CFR 1040)

Atmospheric Correction Range:
- -999.9 ppm to +999.9 ppm, in 0.1 ppm increments

Prism Constant Correction Range:
- -99.9 mm to +99.9 mm, in 0.1 mm increments

Coefficient Factor:
- Meter / Feet
  - International feet: 1 meter = 3.2808398501 ft.
  - US SURVEY feet: 1 meter = 3.280833333 ft.

Electronic Angle Measurement
Method:
- Absolute reading

Detecting system:
- Horizontal: 2 sides
- Vertical: 2 sides

Minimum reading:
- IS 01: 1"/0.5" (0.5 mgon/0.1 mgon, 5 mmil/2 mmil) reading
- IS 03: 5"/1" (1 mgon/0.2 mgon, 20 mmil/5 mmil) reading

Accuracy (Standard deviation based on DIN 18723):
- IS 01:
  - 1" (0.3 mgon)
- IS 03:
  - 3" (1.0 mgon)

Diameter of circle:
- 71 mm

Tilt Correction
Type:
- Automatic vertical and horizontal index

Method:
- Liquid type

Compensating Range:
- ±6'

Correction unit:
- 1" (0.1 mgon)

Computer unit:
- Intel PXA255
Pre-analysis Formulas

Pointing Error

\[ E_{pb} = \text{Error in pointing to the backsight} \]

\[ E_{pf} = \text{Error in pointing to the foresight} \]

\[ d_c = \text{Estimate of how closely the observer can center the cross-hairs on the target} \]

\[ D = \text{The distance to the backsight or the foresight} \]
Pre-analysis Formulas

Pointing Error

\[ \frac{d_c}{D} = \text{Angle in radians} \]

\[ \frac{d_c}{D} \ (206,265) = \text{Angle in Seconds} \]
Pre-analysis Formulas

Pointing Error

\[ E_{pb} = \pm \frac{d_b}{D_b} \quad (206,265) = \text{Pointing Error, Backsite} \]

\[ E_{pf} = \pm \frac{d_f}{D_f} \quad (206,265) = \text{Pointing Error, Foresite} \]
Pointing Error Example

Pointing Error

\[ E_{p_b} = \pm \frac{d_b}{D_b} (206265) \quad d_b = 0.01'; \quad D_b = 350.25 \]

\[ E_{p_b} = \pm \frac{0.01}{350.25} (206,265) = 5.89'' \]
Pointing Error Example

Pointing Error

\[ E_{pf} = \pm \frac{d_f}{D_f} (206,265) \quad d_f = 0.01 \, \text{'}; \quad D_f = 425.36 \]
Pre-analysis Formulas

Pointing Error

Error in one angle turned

\[ E_p = \pm \sqrt{E_{pb}^2 + E_{pf}^2} \]
Pointing Error Example

**Pointing Error**

*Error in one angle turned*

\[ E_p = \pm \sqrt{E_{p_b}^2 + E_{p_f}^2} \]

\[ E_p = \pm \sqrt{5.89^2 + 4.85^2} = 7.63" \]
Pointing Error Example

Pointing Error

Total error in multiple sets turned

\[ \sigma_{\alpha_p} = \pm \frac{E_p \sqrt{2}}{\sqrt{n}} \]

Total Pointing Error

\[ \sigma_{\alpha_p} = \pm \frac{7.63\sqrt{2}}{\sqrt{4}} = 5.40'' \]
Pre-analysis Formulas

Instrument centering error

\[ \sigma_\hat{\theta} = \pm \frac{d_c D_3}{D_f D_b \sqrt{2}} (206,265) = \text{Angle In Seconds} \]

\( d_c = \text{Estimate of how well you can set up over a point} \)

\( D_b = \text{Distance to backsight} \)

\( D_f = \text{Distance to foresight} \)

\( D_3 = \text{Distance from the backsight to the foresight} \)
Instrument Centering Error

Example:

\[
\sigma_{\alpha_i} = \pm \frac{d_c D_3}{D_f D_b \sqrt{2}} (206,265) \\
\]

\[
d_c = 0.005' \quad D_f = 425.36' \\
D_b = 350.25' \quad D_3 = 372.11' \\
\]

\[
\sigma_{\alpha_i} = \pm \frac{(0.005')(372.11')}{(425.36')(350.25')\sqrt{2}} (206,265) = 1.82''
\]
Target Centering Error

\[ d_b + d_f = \text{How well you can set up a target over a point} \]

\[ D_b = \text{Distance to the backsight} \]

\[ D_f = \text{Distance to the foresight} \]

\[ E_{tb} & E_{tf} = \text{Angular error to the backsight and foresight in seconds} \]

\[ \sigma_{\alpha t} = \text{Total angular error} \]
Pre-analysis Formulas

Target Centering Error

\[ E_{tb} = \pm \frac{d_b}{D_b} \text{(206,265)} \]

\[ E_{tf} = \pm \frac{d_f}{D_f} \text{(206,265)} \]
Pre-analysis Formulas

Target Centering Error

\[ \sigma_{\alpha t} = \pm \sqrt{E_{tb}^2 + E_{tf}^2} \]
\[ \frac{E_{t_b}}{D_{t_b}} = \frac{d_{t_b}}{(206,265)} \]

\[ E_{t_b} = 0.005 \]

\[ D_{t_b} = 350.25 \]

\[ d_{t_b} = 0.005 \]

\[ (206,265) = 2.94'' \]

\[ \frac{E_{t_b}}{350.25} = 0.005 \]
Target Centering Error Example

\[ E_{tf} = \frac{d_f}{D_f} (206,265) \]

\[ d_f = 0.005' \]
\[ D_f = 425.36' \]

\[ E_{tf} = \frac{0.005'}{425.36'} (206,265) = 2.42" \]
Target Centering Error Example

\[ \sigma_{\alpha_t} = \pm \sqrt{E_{tb}^2 + E_{tf}^2} \]

\[ E_{tb} = 2.94'' \]
\[ E_{tf} = 2.42'' \]

\[ \sigma_{\alpha t} = \pm \sqrt{2.94^2 + 2.42^2} = 3.81'' \]
Pre-analysis Formulas

Bubble Centering Error

\[
\sigma_{\alpha_b} = \pm \sqrt{\left( f_d \mu \tan \gamma_b \right)^2 + \left( f_d \mu \tan \gamma_f \right)^2} \frac{n}{n}
\]

\(\gamma = \text{Vertical angle to backsight and foresight (not zenith angles)}\)

\(\mu = \text{Bubble sensitivity of the spirit level in seconds}\)
Pre-analysis Formula

Bubble Centering Error

$$\sigma_{\alpha b} = \pm \frac{\sqrt{(f_d \mu \text{Tan} \gamma_b)^2 + (f_d \mu \text{Tan} \gamma_f)^2}}{n}$$

$f_d$ = Estimate of how closely the instrument can be leveled during a set of angles in fractions of divisions of the spirit level

$n$ = Number of angles turned
Bubble Centering Error Example

\[ \sigma_{\alpha\beta} = \pm \sqrt{(f_d \mu \tan \gamma_b)^2 + (f_d \mu \tan \gamma_f)^2} \]

\[ \frac{n}{n} \]

\[ f_d = 0.5 \quad \mu = 10" \quad \gamma_b = 25^\circ30'45" \]

\[ \gamma_f = 32^\circ15'09" \quad n = 4 \]
Bubble Centering Error Example

\[ \sigma_{\alpha_b} = \pm \sqrt{\left(0.5 \times 10 \times \tan 25^\circ 30' 45''\right)^2 + \left(0.5 \times 10 \times \tan 32^\circ 15' 09''\right)^2} \div 4 = 0.99'' \]
Total Random Error

\[ \sigma_{\alpha} = \pm \sqrt{\sigma_r^2 + \sigma_p^2 + \sigma_i^2 + \sigma_t^2 + \sigma_b^2} \]

- \( \sigma_r = \) Reading Error
- \( \sigma_p = \) Pointing Error
- \( \sigma_i = \) Instrument Centering Error
- \( \sigma_t = \) Target Centering Error
- \( \sigma_b = \) Bubble Centering Error
Total Random Error

\[ \sigma_\alpha = \pm \sqrt{0.46^2 + 5.40^2 + 1.82^2 + 3.81^2 + 0.99^2} = 6.93'' \]

**Reading Error** \[ \text{directional} \] = 0.46”

**Pointing Error** = 5.40”

**Instrument Centering Error** = 1.82”

**Target Centering Error** = 3.81”

**Bubble Centering Error** = 0.99”

**Total Angular Error** = 6.93”
Pre-analysis Formulas

**EDM Error**

\[ \sigma_{EDM} = \pm \sqrt{E_m^2 + E_t^2 + E_i^2} \]

- \(E_m = \) (Standard manufacturer error) \((\text{distance})\)
- \(E_t = \) Target centering error in feet
- \(E_i = \) Instrument centering error in feet
Pre-analysis Formulas

EDM Error

\[ \sigma_{EDM} = \pm \sqrt{0.01^2 + 0.005^2 + 0.005^2} = 0.01' \]

\[ E_m = 0.01 \pm [3\text{mm} + (3\text{ppm})(425.36')] \]

\[ E_t = 0.005' \text{ Target Centering} \]

\[ E_i = 0.005' \text{ Instrument Centering} \]
Pre-analysis Formulas

### 18 SPECIFICATIONS

#### Measurement accuracy/Least Count in Measurement/Measurement Time

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measurement mode</th>
<th>Measurement accuracy</th>
<th>Least Count in Measurement</th>
<th>Measurement Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prism mode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine</td>
<td>0.2mm mode</td>
<td>± (2mm +2ppm x D) m.s.e.</td>
<td>0.2mm (0.001ft)</td>
<td>Approx. 3 sec. (Initial 4 sec.)</td>
</tr>
<tr>
<td></td>
<td>1mm mode</td>
<td>1mm (0.005ft)</td>
<td>Approx. 1.2 sec. (Initial 3 sec.)</td>
<td></td>
</tr>
<tr>
<td>Coarse</td>
<td>1mm mode</td>
<td>± (7mm +2ppm x D) m.s.e.</td>
<td>1mm (0.005ft)</td>
<td>Approx. 0.5 sec. (Initial 2.5 sec.)</td>
</tr>
<tr>
<td></td>
<td>10mm mode</td>
<td>± (10mm +2ppm x D) m.s.e.</td>
<td>10mm (0.02ft)</td>
<td>Approx. 0.3 sec. (Initial 2.5 sec.)</td>
</tr>
</tbody>
</table>

D: Measuring distance

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measurement mode</th>
<th>Measurement accuracy</th>
<th>Least Count in Measurement</th>
<th>Measurement Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-prism mode (Diffusing Surface)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine</td>
<td>0.2mm mode</td>
<td>± (5mm) m.s.e.</td>
<td>0.2mm (0.001ft)</td>
<td>Approx. 3 sec. (Initial 4 sec.)</td>
</tr>
<tr>
<td></td>
<td>1mm mode</td>
<td>1mm (0.005ft)</td>
<td>Approx. 1.2 sec. (Initial 3 sec.)</td>
<td></td>
</tr>
<tr>
<td>Coarse</td>
<td>1mm mode</td>
<td>± (10mm) m.s.e.</td>
<td>1mm (0.005ft)</td>
<td>Approx. 0.5 sec. (Initial 2.5 sec.)</td>
</tr>
<tr>
<td></td>
<td>10mm mode</td>
<td>10mm (0.02ft)</td>
<td>Approx. 0.3 sec. (Initial 2.5 sec.)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measurement mode</th>
<th>Measurement accuracy</th>
<th>Least Count in Measurement</th>
<th>Measurement Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-prism long mode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine</td>
<td>1mm mode</td>
<td>± (10mm +10ppm x D) m.s.e.</td>
<td>1mm (0.005ft)</td>
<td>Approx. 1.5–6 sec. (Initial 6–8 sec.)</td>
</tr>
<tr>
<td>Coarse</td>
<td>5mm mode</td>
<td>± (20mm +10ppm x D) m.s.e.</td>
<td>5mm (0.02ft)</td>
<td>Approx. 1–3 sec. (Initial 6–8 sec.)</td>
</tr>
<tr>
<td></td>
<td>10mm mode</td>
<td>± (100mm) m.s.e.</td>
<td>10mm (0.02ft)</td>
<td>Approx. 0.4 sec. (Initial 4–7 sec.)</td>
</tr>
</tbody>
</table>

*1) More than 2m
*2) The initial time will be different by a condition.
*3) Measurement distance: No more than 500m, When Kodak gray card (white surface) is used.
*4) However, when the measurement distance is more than 500 m, or when the reflectance of the measured surface is low, measurement time will become longer.
In Summary

Random errors are part of every measurement taken by a surveyor.

In order to control random errors a surveyor must first understand them.
In Summary

Once the surveyor understands random errors they can place them where they belong.

This is critical because some measurements are better than others.
Weights Of Observations
Some measurements are better than others. This is common knowledge in the surveying profession.
General

Measurements made in good conditions, using good equipment, with proper survey procedures will produce good results.
General

Measurements made in adverse conditions, using poor equipment, and bad surveying procedures will produce bad results.
As a rule, surveyors use good equipment, with good procedures, but can work in poor conditions.
Because of this, the quality of measurements can vary from setup to setup in the same job.
Weighting measurements allows the surveyor to distribute errors of the measurements where the error should go.
General

If one angle in a survey is turned under good conditions and another angle, in the same survey, is turned under bad conditions, weighting measurements allows for putting more of the error in the bad angle than in the good angle.
According to statistical theory, the weight of a measurement is inversely proportional to the variance.
General

The higher the precision of the measurement, the smaller the variance.
General

The smaller the variance, the larger the weight.

OR

The better the measurement, the larger the weight.
The Variance

What is the variance?

Simply put, the variance is the square of the standard deviation.
The standard deviation of a set of measurements can be determined by statistical analysis.
The Variance

This can get complicated in a hurry, and many practicing surveyors tend not to use weights.
The Variance

If weights are not applied to the surveyors measurements, all the measurements are treated the same. This is false!
Weights Of Observations

Weighting measurements does not need to be this complicated

Surveyor can assign weights to their measurements
Weights Of Observations

The worse the measurement, the smaller the weight.

The more precise the measurement, the larger the weight.
Example One

A distance between two points were measured four times.

The first distance was found to be 396.57 feet and was measured with a cloth tape.
The second and third distances were found to be 396.61 and 396.62 feet, and were measured with a steel calibrated tape.
Example One

The fourth distance was found to be 396.64 feet and was measured with an EDM.
Example One

The party chief assigned a weight of one (1) to the first measurement, a weight of two (2) the second and third measurement, and a weight of four (4) to the final measurement.

These values were not based on statistical theory, but based on field judgment.

Low precision, large variance, low weight

High precision, small variance, large weight
Example One

The formula to calculate the weighted mean of the distance is.

\[
\frac{\sum WM}{\sum W} = M_w
\]

\[\sum WM = \text{The sum of the measurements times their weights}\]

\[\sum W = \text{The sum of the weights}\]

\[M_w = \text{The weighted mean distance}\]
Example One

Using the weighted mean formula in our example we have.

\[
\frac{396.57(1) + 396.61(2) + 396.62(2) + 396.64(4)}{1 + 2 + 2 + 4} = 396.62
\]

While the above example shows how to weight measurements, it is not very practical as most distance measurements are made with EDM’s, and the need to weight measurements are rare.
Example Two

A five sided traverse was run with the following results

<table>
<thead>
<tr>
<th>Station</th>
<th>Angle</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>76°46’35”</td>
<td>W = 1</td>
</tr>
<tr>
<td>B</td>
<td>87°15’20”</td>
<td>W = 2</td>
</tr>
<tr>
<td>C</td>
<td>122°10’45”</td>
<td>W = 2</td>
</tr>
<tr>
<td>D</td>
<td>165°58’25”</td>
<td>W = 3</td>
</tr>
<tr>
<td>E</td>
<td>87°48’50”</td>
<td>W = 4</td>
</tr>
<tr>
<td>Total</td>
<td>359°59’55”</td>
<td>12</td>
</tr>
<tr>
<td>Error</td>
<td>00°00’05”</td>
<td></td>
</tr>
</tbody>
</table>
Example Two

Historically, one second would be put into each angle and the error would be eliminated.
Example Two

If this is done, than all of the angles are treated the same, that is, all of the angles are given the same weight.
Example Two

In this case it is the party chief that determines the weights of the angles from the conditions in the field.
Example Two

The party chief assigns the weights of the angles as follows.

Angle “A” is the worst angle.

Angles “B” and “C” are better than “a” but still not real good.

Angle “D” was better than the others but not the best.

Angle “E” was the best angle in the traverse.
Example Two

The weight of angle “A” = 1,
The weights of angles “B” and “C” = 2
The weight of angle “D” = 3
The weight of angle “E” = 4
Example Two

Angle adjustments are made inversely proportional to their weights. The larger the weight of the angle, the smaller the adjustment.
Example Two

The table illustrates how weights are used to distribute errors

<table>
<thead>
<tr>
<th>Station</th>
<th>Measured</th>
<th>Weight</th>
<th>Correction</th>
<th>Numerical Correction</th>
<th>Rounded Correction</th>
<th>Adjusted Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>76°46′35″</td>
<td>1</td>
<td>12X</td>
<td>1.94”</td>
<td>2”</td>
<td>76°46′37″</td>
</tr>
<tr>
<td>B</td>
<td>87°15′20″</td>
<td>2</td>
<td>6X</td>
<td>0.97”</td>
<td>1”</td>
<td>87°15′21″</td>
</tr>
<tr>
<td>C</td>
<td>122°10′45″</td>
<td>2</td>
<td>6X</td>
<td>0.97”</td>
<td>1”</td>
<td>122°10′46″</td>
</tr>
<tr>
<td>D</td>
<td>165°58′25″</td>
<td>3</td>
<td>4X</td>
<td>0.65”</td>
<td>1”</td>
<td>165°58′26″</td>
</tr>
<tr>
<td>E</td>
<td>87°48′50″</td>
<td>4</td>
<td>3X</td>
<td>0.47”</td>
<td>0”</td>
<td>87°48′50″</td>
</tr>
<tr>
<td>Sum</td>
<td>539°59′55″</td>
<td>12</td>
<td>31X</td>
<td>5.00”</td>
<td>5”</td>
<td>540°00′00″</td>
</tr>
</tbody>
</table>

31X=5” X=0.16”
Example Two

To find the correction factor, divide the individual weights into the sum of the weights.

To find the value of “x”, place the sum of the correction factors equal to the error and solve for “x”

To find the numerical correction, multiply the individual correction factor times the value of “x”
Example Two

As seen in the chart, the worst angle received the largest correction (2"), and the best angle had no correction at all.

This is more realistic with what the party chief observed in the field when the angles were turned.
How Weights Can Be Determined

A surveyor can make their own system. One example will be to divide the angles into four categories, as follows.
How Weights Can Be Determined

Category 1 might be one where the setup is on soft ground and the surveyor is having a hard time keeping the instrument level, and their backsite or foresite (or both) is short.

This could receive a weight of one (1)
How Weights Can Be Determined

Category 2: the second example might be where the surveyor is on firm ground but still has a shore backsite and is having a hard time seeing his foresite. This type of angle could receive a weight of two (2)
How Weights Can Be Determined

Category 3: the third example might be where the surveyor is on firm ground, with good sites both front and back, but the conditions are not very good. It could be foggy or there might be a lot of heat waves. This type of angle could receive a weight of three (3).
How Weights Can Be Determined

Category 4: the last example is the best angle. The surveyor is on firm ground with good sites, turning an angle with strong strength of figure. The weather is high overcast skies and about 65°f. This type of angle could receive a weight of four (4).
Conclusion

Many programs, such as star*net, allow you to weight your measurements very easily and with very little practice the surveyor can use weights and put the error where it belongs.
Conclusion

Using weights helps take control of measurements and helps distribute errors in a more realistic manner that can result in a better, more accurate survey.